

In[1]:= Series[x / (1 - Exp[-x]), {x, 0, 6}]

$$\text{Out[1]} = 1 + \frac{x}{2} + \frac{x^2}{12} - \frac{x^4}{720} + \frac{x^6}{30240} + O[x]^7$$

In[2]:= p = 1 + x / 2 + x^2 / 12 - x^4 / 720 + x^6 / 30240

$$\text{Out[2]} = 1 + \frac{x}{2} + \frac{x^2}{12} - \frac{x^4}{720} + \frac{x^6}{30240}$$

In[3]:= Expand[(p /. {x -> a}) \* (p /. {x -> b})]

$$\begin{aligned} \text{Out[3]} = & 1 + \frac{a}{2} + \frac{a^2}{12} - \frac{a^4}{720} + \frac{a^6}{30240} + \frac{b}{2} + \frac{ab}{4} + \frac{a^2b}{24} - \frac{a^4b}{1440} + \frac{a^6b}{60480} + \\ & \frac{b^2}{12} + \frac{ab^2}{24} + \frac{a^2b^2}{144} - \frac{a^4b^2}{8640} + \frac{a^6b^2}{362880} - \frac{b^4}{720} - \frac{ab^4}{1440} - \frac{a^2b^4}{8640} + \frac{a^4b^4}{518400} - \\ & \frac{a^6b^4}{21772800} + \frac{b^6}{30240} + \frac{ab^6}{60480} + \frac{a^2b^6}{362880} - \frac{a^4b^6}{21772800} + \frac{a^6b^6}{914457600} \end{aligned}$$

In[4]:= q = 1 + (1/2) \* (a + b) + (1/12) \* (a^2 + b^2 + 3 \* a \* b) + (1/24) \* (a^2 \* b + a \* b^2) - (1/720) \* (a^4 - 5 \* a^2 \* b^2 + b^4) - (1/1440) \* (a^4 \* b + a \* b^4) + (1/30240) \* (a^6 - (30240/8640) \* a^4 \* b^2 - (30240/8640) \* a^2 \* b^4 + b^6)

$$\begin{aligned} \text{Out[4]} = & 1 + \frac{a+b}{2} + \frac{1}{12} (a^2 + 3ab + b^2) + \frac{1}{24} (a^2b + ab^2) + \\ & \frac{1}{720} (-a^4 + 5a^2b^2 - b^4) + \frac{-a^4b - ab^4}{1440} + \frac{a^6 - \frac{7a^4b^2}{2} - \frac{7a^2b^4}{2} + b^6}{30240} \end{aligned}$$

In[5]:= q1 = Expand[q /. {a -> c, b -> d}]

$$\begin{aligned} \text{Out[5]} = & 1 + \frac{c}{2} + \frac{c^2}{12} - \frac{c^4}{720} + \frac{c^6}{30240} + \frac{d}{2} + \frac{cd}{4} + \frac{c^2d}{24} - \frac{c^4d}{1440} + \\ & \frac{d^2}{12} + \frac{cd^2}{24} + \frac{c^2d^2}{144} - \frac{c^4d^2}{8640} - \frac{d^4}{720} - \frac{cd^4}{1440} - \frac{c^2d^4}{8640} + \frac{d^6}{30240} \end{aligned}$$

In[6]:= q2 = Expand[q /. {a -> e, b -> f}]

$$\begin{aligned} \text{Out[6]} = & 1 + \frac{e}{2} + \frac{e^2}{12} - \frac{e^4}{720} + \frac{e^6}{30240} + \frac{f}{2} + \frac{ef}{4} + \frac{e^2f}{24} - \frac{e^4f}{1440} + \\ & \frac{f^2}{12} + \frac{ef^2}{24} + \frac{e^2f^2}{144} - \frac{e^4f^2}{8640} - \frac{f^4}{720} - \frac{ef^4}{1440} - \frac{e^2f^4}{8640} + \frac{f^6}{30240} \end{aligned}$$

In[7]:= Expand[q \* q1 \* q2]

$$\begin{aligned} \text{Out[7]} = & 1 + \frac{a}{2} + \frac{a^2}{12} - \frac{a^4}{720} + \frac{a^6}{30240} + \frac{b}{2} + \frac{ab}{4} + \dots + \frac{b^2d^6f^6}{10973491200} + \frac{ab^2d^6f^6}{21946982400} + \\ & \frac{a^2b^2d^6f^6}{131681894400} - \frac{a^4b^2d^6f^6}{7900913664000} - \frac{b^4d^6f^6}{658409472000} - \frac{ab^4d^6f^6}{1316818944000} - \frac{a^2b^4d^6f^6}{7900913664000} + \frac{b^6d^6f^6}{27653197824000} \end{aligned}$$

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In[8]:= SeriesCoefficient[%7, {a, 0, 6}, {b, 0, 0}, {c, 0, 0}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]

$$\text{Out[8]} = \frac{1}{30240}$$

```
In[9]:= SeriesCoefficient[%7, {a, 0, 5},
      {b, 0, 1}, {c, 0, 0}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[9]= 0
```

```
In[10]:= SeriesCoefficient[%7, {a, 0, 4},
      {b, 0, 2}, {c, 0, 0}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[10]=

$$-\frac{1}{8640}$$

```

```
In[11]:= SeriesCoefficient[%7, {a, 0, 4},
      {b, 0, 1}, {c, 0, 1}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[11]=

$$-\frac{1}{2880}$$

```

```
In[12]:= SeriesCoefficient[%7, {a, 0, 3},
      {b, 0, 3}, {c, 0, 0}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[12]=
0
```

```
In[13]:= SeriesCoefficient[%7, {a, 0, 3},
      {b, 0, 2}, {c, 0, 1}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[13]=
0
```

```
In[14]:= SeriesCoefficient[%7, {a, 0, 3},
      {b, 0, 1}, {c, 0, 1}, {d, 0, 1}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[14]=
0
```

```
In[15]:= SeriesCoefficient[%7, {a, 0, 2},
      {b, 0, 2}, {c, 0, 2}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[15]=

$$\frac{1}{1728}$$

```

```
In[16]:= SeriesCoefficient[%7, {a, 0, 2},
      {b, 0, 2}, {c, 0, 1}, {d, 0, 1}, {e, 0, 0}, {f, 0, 0}]
```

```
Out[16]=

$$\frac{1}{576}$$

```

```
In[17]:= SeriesCoefficient[%7, {a, 0, 2},
      {b, 0, 1}, {c, 0, 1}, {d, 0, 1}, {e, 0, 1}, {f, 0, 0}]
```

```
Out[17]=

$$\frac{1}{192}$$

```

In[18]:= **SeriesCoefficient**[%7, {a, 0, 1},  
 {b, 0, 1}, {c, 0, 1}, {d, 0, 1}, {e, 0, 1}, {f, 0, 1}]

Out[18]=  

$$\frac{1}{64}$$

In[19]:= **AugmentedSymmetricPolynomial**[{6}, {a, b, c, d, e, f}]

Out[19]=  

$$a^6 + b^6 + c^6 + d^6 + e^6 + f^6$$

In[20]:= **AugmentedSymmetricPolynomial**[{2, 4}, {a, b, c, d, e, f}]

Out[20]=  

$$\begin{aligned} & a^4 b^2 + a^2 b^4 + a^4 c^2 + b^4 c^2 + a^2 c^4 + b^2 c^4 + a^4 d^2 + b^4 d^2 + c^4 d^2 + \\ & a^2 d^4 + b^2 d^4 + c^2 d^4 + a^4 e^2 + b^4 e^2 + c^4 e^2 + d^4 e^2 + a^2 e^4 + b^2 e^4 + c^2 e^4 + \\ & d^2 e^4 + a^4 f^2 + b^4 f^2 + c^4 f^2 + d^4 f^2 + e^4 f^2 + a^2 f^4 + b^2 f^4 + c^2 f^4 + d^2 f^4 + e^2 f^4 \end{aligned}$$

In[21]:= **AugmentedSymmetricPolynomial**[{1, 1, 4}, {a, b, c, d, e, f}]

Out[21]=  

$$\begin{aligned} & 2 a^4 b c + 2 a b^4 c + 2 a b c^4 + 2 a^4 b d + 2 a b^4 d + 2 a^4 c d + 2 b^4 c d + 2 a c^4 d + 2 b c^4 d + \\ & 2 a b d^4 + 2 a c d^4 + 2 b c d^4 + 2 a^4 b e + 2 a b^4 e + 2 a^4 c e + 2 b^4 c e + 2 a c^4 e + 2 b c^4 e + \\ & 2 a^4 d e + 2 b^4 d e + 2 c^4 d e + 2 a d^4 e + 2 b d^4 e + 2 c d^4 e + 2 a b e^4 + 2 a c e^4 + 2 b c e^4 + \\ & 2 a d e^4 + 2 b d e^4 + 2 c d e^4 + 2 a^4 b f + 2 a b^4 f + 2 a^4 c f + 2 b^4 c f + 2 a c^4 f + \\ & 2 b c^4 f + 2 a^4 d f + 2 b^4 d f + 2 c^4 d f + 2 a d^4 f + 2 b d^4 f + 2 c d^4 f + 2 a^4 e f + \\ & 2 b^4 e f + 2 c^4 e f + 2 d^4 e f + 2 a e^4 f + 2 b e^4 f + 2 c e^4 f + 2 d e^4 f + 2 a b f^4 + \\ & 2 a c f^4 + 2 b c f^4 + 2 a d f^4 + 2 b d f^4 + 2 c d f^4 + 2 a e f^4 + 2 b e f^4 + 2 c e f^4 + 2 d e f^4 \end{aligned}$$

In[22]:= **AugmentedSymmetricPolynomial**[{2, 2, 2}, {a, b, c, d, e, f}]

Out[22]=  

$$\begin{aligned} & 6 a^2 b^2 c^2 + 6 a^2 b^2 d^2 + 6 a^2 c^2 d^2 + 6 b^2 c^2 d^2 + 6 a^2 b^2 e^2 + 6 a^2 c^2 e^2 + \\ & 6 b^2 c^2 e^2 + 6 a^2 d^2 e^2 + 6 b^2 d^2 e^2 + 6 c^2 d^2 e^2 + 6 a^2 b^2 f^2 + 6 a^2 c^2 f^2 + 6 b^2 c^2 f^2 + \\ & 6 a^2 d^2 f^2 + 6 b^2 d^2 f^2 + 6 c^2 d^2 f^2 + 6 a^2 e^2 f^2 + 6 b^2 e^2 f^2 + 6 c^2 e^2 f^2 + 6 d^2 e^2 f^2 \end{aligned}$$

In[23]:= **AugmentedSymmetricPolynomial**[{1, 1, 2, 2}, {a, b, c, d, e, f}]

Out[23]=  

$$\begin{aligned} & 4 a^2 b^2 c d + 4 a^2 b c^2 d + 4 a b^2 c^2 d + 4 a^2 b c d^2 + 4 a b^2 c d^2 + 4 a b c^2 d^2 + 4 a^2 b^2 c e + \\ & 4 a^2 b c^2 e + 4 a b^2 c^2 e + 4 a^2 b^2 d e + 4 a^2 c^2 d e + 4 b^2 c^2 d e + 4 a^2 b d^2 e + \\ & 4 a b^2 d^2 e + 4 a^2 c d^2 e + 4 b^2 c d^2 e + 4 a c^2 d^2 e + 4 b c^2 d^2 e + 4 a^2 b c e^2 + 4 a b^2 c e^2 + \\ & 4 a b c^2 e^2 + 4 a^2 b d e^2 + 4 a b^2 d e^2 + 4 a^2 c d e^2 + 4 b^2 c d e^2 + 4 a c^2 d e^2 + 4 b c^2 d e^2 + \\ & 4 a b d^2 e^2 + 4 a c d^2 e^2 + 4 b c d^2 e^2 + 4 a^2 b^2 c f + 4 a^2 b c^2 f + 4 a b^2 c^2 f + 4 a^2 b^2 d f + \\ & 4 a^2 c^2 d f + 4 b^2 c^2 d f + 4 a^2 b d^2 f + 4 a b^2 d^2 f + 4 a^2 c d^2 f + 4 b^2 c d^2 f + 4 a c^2 d^2 f + \\ & 4 b c^2 d^2 f + 4 a^2 b^2 e f + 4 a^2 c^2 e f + 4 b^2 c^2 e f + 4 a^2 d^2 e f + 4 b^2 d^2 e f + 4 c^2 d^2 e f + \\ & 4 a^2 b e^2 f + 4 a b^2 e^2 f + 4 a^2 c e^2 f + 4 b^2 c e^2 f + 4 a c^2 e^2 f + 4 b c^2 e^2 f + 4 a^2 d e^2 f + \\ & 4 b^2 d e^2 f + 4 c^2 d e^2 f + 4 a d^2 e^2 f + 4 b d^2 e^2 f + 4 c d^2 e^2 f + 4 a^2 b c f^2 + 4 a b^2 c f^2 + \\ & 4 a b c^2 f^2 + 4 a^2 b d f^2 + 4 a b^2 d f^2 + 4 a^2 c d f^2 + 4 b^2 c d f^2 + 4 a c^2 d f^2 + 4 b c^2 d f^2 + \\ & 4 a b d^2 f^2 + 4 a c d^2 f^2 + 4 b c d^2 f^2 + 4 a^2 b e f^2 + 4 a b^2 e f^2 + 4 a^2 c e f^2 + 4 b^2 c e f^2 + \\ & 4 a c^2 e f^2 + 4 b c^2 e f^2 + 4 a^2 d e f^2 + 4 b^2 d e f^2 + 4 c^2 d e f^2 + 4 a d^2 e f^2 + 4 b d^2 e f^2 + \\ & 4 c d^2 e f^2 + 4 a b e^2 f^2 + 4 a c e^2 f^2 + 4 b c e^2 f^2 + 4 a d e^2 f^2 + 4 b d e^2 f^2 + 4 c d e^2 f^2 \end{aligned}$$

In[24]:= **AugmentedSymmetricPolynomial**[{1, 1, 1, 1, 2}, {a, b, c, d, e, f}]

Out[24]=

$$24 a^2 b c d e + 24 a b^2 c d e + 24 a b c^2 d e + 24 a b c d^2 e + 24 a b c d e^2 + 24 a^2 b c d f + 24 a b^2 c d f + 24 a b c^2 d f + 24 a b c d^2 f + 24 a^2 b c e f + 24 a b^2 c e f + 24 a b c^2 e f + 24 a^2 b d e f + 24 a b^2 d e f + 24 a^2 c d e f + 24 b^2 c d e f + 24 a c^2 d e f + 24 b c^2 d e f + 24 a b d^2 e f + 24 a c d^2 e f + 24 b c d^2 e f + 24 a b c e^2 f + 24 a b d e^2 f + 24 a c d e^2 f + 24 b c d e^2 f + 24 a b c d f^2 + 24 a b c e f^2 + 24 a b d e f^2 + 24 a c d e f^2 + 24 b c d e f^2$$

In[25]:= **AugmentedSymmetricPolynomial**[{1, 1, 1, 1, 1, 1}, {a, b, c, d, e, f}]

Out[25]=

$$720 a b c d e f$$

In[26]:= **SymmetricReduction**[%19, {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]

Out[26]=

$$\{c1^6 - 6 c1^4 c2 + 9 c1^2 c2^2 - 2 c2^3 + 6 c1^3 c3 - 12 c1 c2 c3 + 3 c3^2 - 6 c1^2 c4 + 6 c2 c4 + 6 c1 c5 - 6 c6, 0\}$$

In[27]:= **t6 = First**[%26]

Out[27]=

$$c1^6 - 6 c1^4 c2 + 9 c1^2 c2^2 - 2 c2^3 + 6 c1^3 c3 - 12 c1 c2 c3 + 3 c3^2 - 6 c1^2 c4 + 6 c2 c4 + 6 c1 c5 - 6 c6$$

In[28]:= **SymmetricReduction**[%20, {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]

Out[28]=

$$\{c1^2 c2^2 - 2 c2^3 - 2 c1^3 c3 + 4 c1 c2 c3 - 3 c3^2 + 2 c1^2 c4 + 2 c2 c4 - 6 c1 c5 + 6 c6, 0\}$$

In[29]:= **t42 = First**[%28]

Out[29]=

$$c1^2 c2^2 - 2 c2^3 - 2 c1^3 c3 + 4 c1 c2 c3 - 3 c3^2 + 2 c1^2 c4 + 2 c2 c4 - 6 c1 c5 + 6 c6$$

In[30]:= **SymmetricReduction**[(1/2) \* (%21), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]

Out[30]=

$$\{c1^3 c3 - 3 c1 c2 c3 + 3 c3^2 - c1^2 c4 + 2 c2 c4 + c1 c5 - 6 c6, 0\}$$

In[31]:= **t411 = First**[%30]

Out[31]=

$$c1^3 c3 - 3 c1 c2 c3 + 3 c3^2 - c1^2 c4 + 2 c2 c4 + c1 c5 - 6 c6$$

In[32]:= **SymmetricReduction**[(1/6) \* (%22), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]

Out[32]=

$$\{c3^2 - 2 c2 c4 + 2 c1 c5 - 2 c6, 0\}$$

In[33]:= **t222 = First**[%32]

Out[33]=

$$c3^2 - 2 c2 c4 + 2 c1 c5 - 2 c6$$

In[34]:= **SymmetricReduction**[(1/4) \* (%23), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]

Out[34]=

$$\{c2 c4 - 4 c1 c5 + 9 c6, 0\}$$

```

In[35]:= t2211 = First[%34]
Out[35]=
  c2 c4 - 4 c1 c5 + 9 c6

In[36]:= SymmetricReduction[(1/24) * (%24), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]
Out[36]=
  {c1 c5 - 6 c6, 0}

In[37]:= t21111 = First[%36]
Out[37]=
  c1 c5 - 6 c6

In[38]:= SymmetricReduction[(1/720) * (%25), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]
Out[38]=
  {c6, 0}

In[39]:= t111111 = First[%38]
Out[39]=
  c6

In[40]:= Expand[t6 - (30 240 / 8640) * t42 - (30 240 / 2880) * t411 + (30 240 / 1728) * t222 +
  (30 240 / 576) * t2211 + (30 240 / 192) * t21111 + (30 240 / 64) * t111111]
Out[40]=

$$c1^6 - 6 c1^4 c2 + \frac{11 c1^2 c2^2}{2} + 5 c2^3 + \frac{5 c1^3 c3}{2} +$$


$$\frac{11 c1 c2 c3}{2} - \frac{c3^2}{2} - \frac{5 c1^2 c4}{2} - \frac{9 c2 c4}{2} - c1 c5 + c6$$


In[41]:= SeriesCoefficient[%7, {a, 0, 5},
  {b, 0, 0}, {c, 0, 0}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
Out[41]=
  0

In[42]:= SeriesCoefficient[%7, {a, 0, 4},
  {b, 0, 1}, {c, 0, 0}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
Out[42]=

$$-\frac{1}{1440}$$


In[43]:= SeriesCoefficient[%7, {a, 0, 3},
  {b, 0, 2}, {c, 0, 0}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
Out[43]=
  0

In[44]:= SeriesCoefficient[%7, {a, 0, 3},
  {b, 0, 1}, {c, 0, 1}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]
Out[44]=
  0

```

In[45]:= **SeriesCoefficient**[%7, {a, 0, 2},  
 {b, 0, 2}, {c, 0, 1}, {d, 0, 0}, {e, 0, 0}, {f, 0, 0}]

Out[45]=  

$$\frac{1}{288}$$

In[46]:= **SeriesCoefficient**[%7, {a, 0, 2},  
 {b, 0, 1}, {c, 0, 1}, {d, 0, 1}, {e, 0, 0}, {f, 0, 0}]

Out[46]=  

$$\frac{1}{96}$$

In[47]:= **SeriesCoefficient**[%7, {a, 0, 1},  
 {b, 0, 1}, {c, 0, 1}, {d, 0, 1}, {e, 0, 1}, {f, 0, 0}]

Out[47]=  

$$\frac{1}{32}$$

In[48]:= **AugmentedSymmetricPolynomial**[{1, 4}, {a, b, c, d, e, f}]

Out[48]=  

$$a^4 b + a b^4 + a^4 c + b^4 c + a c^4 + b c^4 + a^4 d + b^4 d + c^4 d + a d^4 + b d^4 + c d^4 + a^4 e + b^4 e + c^4 e + d^4 e + a e^4 + b e^4 + c e^4 + d e^4 + a^4 f + b^4 f + c^4 f + d^4 f + e^4 f + a f^4 + b f^4 + c f^4 + d f^4 + e f^4$$

In[49]:= **AugmentedSymmetricPolynomial**[{1, 2, 2}, {a, b, c, d, e, f}]

Out[49]=  

$$2 a^2 b^2 c + 2 a^2 b c^2 + 2 a b^2 c^2 + 2 a^2 b^2 d + 2 a^2 c^2 d + 2 b^2 c^2 d + 2 a^2 b d^2 + 2 a b^2 d^2 + 2 a^2 c d^2 + 2 b^2 c d^2 + 2 a c^2 d^2 + 2 b c^2 d^2 + 2 a^2 b^2 e + 2 a^2 c^2 e + 2 b^2 c^2 e + 2 a^2 d^2 e + 2 b^2 d^2 e + 2 c^2 d^2 e + 2 a^2 b e^2 + 2 a b^2 e^2 + 2 a^2 c e^2 + 2 b^2 c e^2 + 2 a c^2 e^2 + 2 b c^2 e^2 + 2 a^2 d e^2 + 2 b^2 d e^2 + 2 c^2 d e^2 + 2 a d^2 e^2 + 2 b d^2 e^2 + 2 c d^2 e^2 + 2 a^2 b^2 f + 2 a^2 c^2 f + 2 b^2 c^2 f + 2 a^2 d^2 f + 2 b^2 d^2 f + 2 c^2 d^2 f + 2 a^2 e^2 f + 2 b^2 e^2 f + 2 c^2 e^2 f + 2 d^2 e^2 f + 2 a^2 b f^2 + 2 a b^2 f^2 + 2 a^2 c f^2 + 2 b^2 c f^2 + 2 a c^2 f^2 + 2 b c^2 f^2 + 2 a^2 d f^2 + 2 b^2 d f^2 + 2 c^2 d f^2 + 2 a d^2 f^2 + 2 b d^2 f^2 + 2 c d^2 f^2 + 2 a^2 e f^2 + 2 b^2 e f^2 + 2 c^2 e f^2 + 2 d^2 e f^2 + 2 a e^2 f^2 + 2 b e^2 f^2 + 2 c e^2 f^2 + 2 d e^2 f^2$$

In[50]:= **AugmentedSymmetricPolynomial**[{1, 1, 1, 2}, {a, b, c, d, e, f}]

Out[50]=  

$$6 a^2 b c d + 6 a b^2 c d + 6 a b c^2 d + 6 a b c d^2 + 6 a^2 b c e + 6 a b^2 c e + 6 a b c^2 e + 6 a^2 b d e + 6 a b^2 d e + 6 a^2 c d e + 6 b^2 c d e + 6 a c^2 d e + 6 b c^2 d e + 6 a b d^2 e + 6 a c d^2 e + 6 b c d^2 e + 6 a b c e^2 + 6 a b d e^2 + 6 a c d e^2 + 6 b c d e^2 + 6 a^2 b c f + 6 a b^2 c f + 6 a b c^2 f + 6 a^2 b d f + 6 a b^2 d f + 6 a^2 c d f + 6 b^2 c d f + 6 a c^2 d f + 6 b c^2 d f + 6 a b d^2 f + 6 a c d^2 f + 6 b c d^2 f + 6 a^2 b e f + 6 a b^2 e f + 6 a^2 c e f + 6 b^2 c e f + 6 a c^2 e f + 6 b c^2 e f + 6 a^2 d e f + 6 b^2 d e f + 6 c^2 d e f + 6 a d^2 e f + 6 b d^2 e f + 6 c d^2 e f + 6 a b e^2 f + 6 a c e^2 f + 6 b c e^2 f + 6 a d e^2 f + 6 b d e^2 f + 6 c d e^2 f + 6 a b c f^2 + 6 a b d f^2 + 6 a c d f^2 + 6 b c d f^2 + 6 a b e f^2 + 6 a c e f^2 + 6 b c e f^2 + 6 a d e f^2 + 6 b d e f^2 + 6 c d e f^2$$

In[51]:= **AugmentedSymmetricPolynomial**[{1, 1, 1, 1, 1}, {a, b, c, d, e, f}]

Out[51]=  

$$120 a b c d e + 120 a b c d f + 120 a b c e f + 120 a b d e f + 120 a c d e f + 120 b c d e f$$

```

In[52]:= SymmetricReduction[%48, {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]
Out[52]=

$$\{c1^3 c2 - 3 c1 c2^2 - c1^2 c3 + 5 c2 c3 + c1 c4 - 5 c5, 0\}$$


In[53]:= t41 = First[%52]
Out[53]=

$$c1^3 c2 - 3 c1 c2^2 - c1^2 c3 + 5 c2 c3 + c1 c4 - 5 c5$$


In[54]:= SymmetricReduction[(1/2) * (%49), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]
Out[54]=

$$\{c2 c3 - 3 c1 c4 + 5 c5, 0\}$$


In[55]:= t221 = First[%54]
Out[55]=

$$c2 c3 - 3 c1 c4 + 5 c5$$


In[56]:= SymmetricReduction[(1/6) * (%50), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]
Out[56]=

$$\{c1 c4 - 5 c5, 0\}$$


In[57]:= t2111 = First[%56]
Out[57]=

$$c1 c4 - 5 c5$$


In[58]:= SymmetricReduction[(1/120) * (%51), {a, b, c, d, e, f}, {c1, c2, c3, c4, c5, c6}]
Out[58]=

$$\{c5, 0\}$$


In[59]:= t11111 = First[%58]
Out[59]=

$$c5$$


In[60]:= Expand[-t41 + (1440/288) * t221 + (1440/96) * t2111 + (1440/32) * t11111]
Out[60]=

$$-c1^3 c2 + 3 c1 c2^2 + c1^2 c3 - c1 c4$$


In[61]:= td = 1 + (1/2) * c1 * t + (1/12) * (c1^2 + c2) * t^2 + (1/24) * c1 * c2 * t^3 -
(1/720) (c1^4 - 4 * c1^2 * c2 - 3 * c2^2 - c1 * c3 + c4) * t^4 +
(1/1440) * (%60) * t^5 + (1/30240) * (%40) * t^6
Out[61]=

$$1 + \frac{c1 t}{2} + \frac{1}{12} (c1^2 + c2) t^2 + \frac{1}{24} c1 c2 t^3 -$$


$$\frac{1}{720} (c1^4 - 4 c1^2 c2 - 3 c2^2 - c1 c3 + c4) t^4 + \frac{(-c1^3 c2 + 3 c1 c2^2 + c1^2 c3 - c1 c4) t^5}{1440} +$$


$$\frac{(c1^6 - 6 c1^4 c2 + \frac{11 c1^2 c2^2}{2} + 5 c2^3 + \frac{5 c1^3 c3}{2} + \frac{11 c1 c2 c3}{2} - \frac{c3^2}{2} - \frac{5 c1^2 c4}{2} - \frac{9 c2 c4}{2} - c1 c5 + c6) t^6}{30240}$$


In[62]:= AugmentedSymmetricPolynomial[{6}, {a, b, c, d, e, f}]
Out[62]=

$$a^6 + b^6 + c^6 + d^6 + e^6 + f^6$$


```

In[63]:= **SymmetricReduction**[%62, {a, b, c, d, e, f}, {d1, d2, d3, d4, d5, d6}]

Out[63]=

$$\{d1^6 - 6 d1^4 d2 + 9 d1^2 d2^2 - 2 d2^3 + 6 d1^3 d3 - 12 d1 d2 d3 + 3 d3^2 - 6 d1^2 d4 + 6 d2 d4 + 6 d1 d5 - 6 d6, 0\}$$

In[64]:= **s6 = First**[%63]

Out[64]=

$$d1^6 - 6 d1^4 d2 + 9 d1^2 d2^2 - 2 d2^3 + 6 d1^3 d3 - 12 d1 d2 d3 + 3 d3^2 - 6 d1^2 d4 + 6 d2 d4 + 6 d1 d5 - 6 d6$$

In[65]:= **AugmentedSymmetricPolynomial**[%5], {a, b, c, d, e, f}]

Out[65]=

$$a^5 + b^5 + c^5 + d^5 + e^5 + f^5$$

In[66]:= **SymmetricReduction**[%65, {a, b, c, d, e, f}, {d1, d2, d3, d4, d5, d6}]

Out[66]=

$$\{d1^5 - 5 d1^3 d2 + 5 d1 d2^2 + 5 d1^2 d3 - 5 d2 d3 - 5 d1 d4 + 5 d5, 0\}$$

In[67]:= **s5 = First**[%66]

Out[67]=

$$d1^5 - 5 d1^3 d2 + 5 d1 d2^2 + 5 d1^2 d3 - 5 d2 d3 - 5 d1 d4 + 5 d5$$

In[68]:= **ch = 6 + d1 \* t + (1/2) \* (d1^2 - 2 \* d2) \* t^2 + (1/6) \* (d1^3 - 3 \* d1 \* d2 + 3 \* d3) \* t^3 + (1/24) \* (d1^4 - 4 \* d1^2 \* d2 + 4 \* d1 \* d3 + 2 \* d2^2 - 4 \* d4) \* t^4 + (1/120) \* s5 \* t^5 + (1/720) \* s6 \* t^6**

Out[68]=

$$6 + d1 t + \frac{1}{2} (d1^2 - 2 d2) t^2 + \frac{1}{6} (d1^3 - 3 d1 d2 + 3 d3) t^3 + \frac{1}{24} (d1^4 - 4 d1^2 d2 + 2 d2^2 + 4 d1 d3 - 4 d4) t^4 + \frac{1}{120} (d1^5 - 5 d1^3 d2 + 5 d1 d2^2 + 5 d1^2 d3 - 5 d2 d3 - 5 d1 d4 + 5 d5) t^5 + \frac{1}{720} (d1^6 - 6 d1^4 d2 + 9 d1^2 d2^2 - 2 d2^3 + 6 d1^3 d3 - 12 d1 d2 d3 + 3 d3^2 - 6 d1^2 d4 + 6 d2 d4 + 6 d1 d5 - 6 d6) t^6$$

In[69]:= **Expand**[ch \* td]

Out[69]=

$$6 + 3 c1 t + d1 t + \frac{c1^2 t^2}{2} + \frac{c2 t^2}{2} + \frac{1}{2} c1 d1 t^2 + \frac{d1^2 t^2}{2} - d2 t^2 + \frac{1}{4} c1 c2 t^3 + \frac{1}{12} c1^2 d1 t^3 + \frac{1}{12} c2 d1 t^3 + \frac{1}{4} c1 d1^2 t^3 + \frac{d1^3 t^3}{6} - \frac{1}{2} c1 d2 t^3 - \frac{1}{2} d1 d2 t^3 + \frac{d3 t^3}{2} - \frac{c1^4 t^4}{120} + \frac{1}{30} c1^2 c2 t^4 + \frac{c2^2 t^4}{40} + \frac{1}{120} c1 c3 t^4 - \frac{c4 t^4}{120} + \frac{1}{24} c1 c2 d1 t^4 + \frac{1}{24} c1^2 d1^2 t^4 + \frac{1}{24} c2 d1^2 t^4 + \frac{1}{12} c1 d1^3 t^4 + \frac{d1^4 t^4}{24} - \frac{1}{12} c1^2 d2 t^4 - \frac{1}{12} c2 d2 t^4 - \frac{1}{4} c1 d1 d2 t^4 - \frac{1}{6} d1^2 d2 t^4 + \frac{d2^2 t^4}{12} + \frac{1}{4} c1 d3 t^4 + \frac{1}{6} d1 d3 t^4 - \frac{d4 t^4}{6} - \frac{1}{240} c1^3 c2 t^5 + \frac{1}{80} c1 c2^2 t^5 + \frac{1}{240} c1^2 c3 t^5 - \frac{1}{240} c1 c4 t^5 - \frac{1}{720} c1^4 d1 t^5 + \frac{1}{180} c1^2 c2 d1 t^5 + \frac{1}{240} c2^2 d1 t^5 +$$



$$\begin{aligned}
& \frac{1}{720} c_1 c_3 d_1 t^5 - \frac{1}{720} c_4 d_1 t^5 + \frac{1}{48} c_1 c_2 d_1^2 t^5 + \frac{1}{72} c_1^2 d_1^3 t^5 + \frac{1}{72} c_2 d_1^3 t^5 + \\
& \frac{1}{48} c_1 d_1^4 t^5 + \frac{d_1^5 t^5}{120} - \frac{1}{24} c_1 c_2 d_2 t^5 - \frac{1}{24} c_1^2 d_1 d_2 t^5 - \frac{1}{24} c_2 d_1 d_2 t^5 - \\
& \frac{1}{12} c_1 d_1^2 d_2 t^5 - \frac{1}{24} d_1^3 d_2 t^5 + \frac{1}{24} c_1 d_2^2 t^5 + \frac{1}{24} d_1 d_2^2 t^5 + \frac{1}{24} c_1^2 d_3 t^5 + \frac{1}{24} c_2 d_3 t^5 + \\
& \frac{1}{12} c_1 d_1 d_3 t^5 + \frac{1}{24} d_1^2 d_3 t^5 - \frac{1}{24} d_2 d_3 t^5 - \frac{1}{12} c_1 d_4 t^5 - \frac{1}{24} d_1 d_4 t^5 + \frac{d_5 t^5}{24} + \\
& \frac{c_1^6 t^6}{5040} - \frac{1}{840} c_1^4 c_2 t^6 + \frac{11 c_1^2 c_2^2 t^6}{10080} + \frac{c_2^3 t^6}{1008} + \frac{c_1^3 c_3 t^6}{2016} + \frac{11 c_1 c_2 c_3 t^6}{10080} - \frac{c_3^2 t^6}{10080} - \\
& \frac{c_1^2 c_4 t^6}{2016} - \frac{c_2 c_4 t^6}{1120} - \frac{c_1 c_5 t^6}{5040} + \frac{c_6 t^6}{5040} - \frac{c_1^3 c_2 d_1 t^6}{1440} + \frac{1}{480} c_1 c_2^2 d_1 t^6 + \frac{c_1^2 c_3 d_1 t^6}{1440} - \\
& \frac{c_1 c_4 d_1 t^6}{1440} - \frac{c_1^4 d_1^2 t^6}{1440} + \frac{1}{360} c_1^2 c_2 d_1^2 t^6 + \frac{1}{480} c_2^2 d_1^2 t^6 + \frac{c_1 c_3 d_1^2 t^6}{1440} - \\
& \frac{c_4 d_1^2 t^6}{1440} + \frac{1}{144} c_1 c_2 d_1^3 t^6 + \frac{1}{288} c_1^2 d_1^4 t^6 + \frac{1}{288} c_2 d_1^4 t^6 + \frac{1}{240} c_1 d_1^5 t^6 + \frac{d_1^6 t^6}{720} + \\
& \frac{1}{720} c_1^4 d_2 t^6 - \frac{1}{180} c_1^2 c_2 d_2 t^6 - \frac{1}{240} c_2^2 d_2 t^6 - \frac{1}{720} c_1 c_3 d_2 t^6 + \frac{1}{720} c_4 d_2 t^6 - \\
& \frac{1}{48} c_1 c_2 d_1 d_2 t^6 - \frac{1}{72} c_1^2 d_1^2 d_2 t^6 - \frac{1}{72} c_2 d_1^2 d_2 t^6 - \frac{1}{48} c_1 d_1^3 d_2 t^6 - \frac{1}{120} d_1^4 d_2 t^6 + \\
& \frac{1}{144} c_1^2 d_2^2 t^6 + \frac{1}{144} c_2 d_2^2 t^6 + \frac{1}{48} c_1 d_1 d_2^2 t^6 + \frac{1}{80} d_1^2 d_2^2 t^6 - \frac{d_2^3 t^6}{360} + \frac{1}{48} c_1 c_2 d_3 t^6 + \\
& \frac{1}{72} c_1^2 d_1 d_3 t^6 + \frac{1}{72} c_2 d_1 d_3 t^6 + \frac{1}{48} c_1 d_1^2 d_3 t^6 + \frac{1}{120} d_1^3 d_3 t^6 - \frac{1}{48} c_1 d_2 d_3 t^6 - \\
& \frac{1}{60} d_1 d_2 d_3 t^6 + \frac{d_3^2 t^6}{240} - \frac{1}{72} c_1^2 d_4 t^6 - \frac{1}{72} c_2 d_4 t^6 - \frac{1}{48} c_1 d_1 d_4 t^6 - \frac{1}{120} d_1^2 d_4 t^6 + \\
& \frac{1}{120} d_2 d_4 t^6 + \frac{1}{48} c_1 d_5 t^6 + \frac{1}{120} d_1 d_5 t^6 - \frac{d_6 t^6}{120} + \frac{c_1^6 d_1 t^7}{30240} - \frac{c_1^4 c_2 d_1 t^7}{5040} + \\
& \frac{11 c_1^2 c_2^2 d_1 t^7}{60480} + \frac{c_2^3 d_1 t^7}{6048} + \frac{c_1^3 c_3 d_1 t^7}{12096} + \frac{11 c_1 c_2 c_3 d_1 t^7}{60480} - \frac{c_3^2 d_1 t^7}{60480} - \\
& \frac{c_1^2 c_4 d_1 t^7}{12096} - \frac{c_2 c_4 d_1 t^7}{6720} - \frac{c_1 c_5 d_1 t^7}{30240} + \frac{c_6 d_1 t^7}{30240} - \frac{c_1^3 c_2 d_1^2 t^7}{2880} + \frac{1}{960} c_1 c_2^2 d_1^2 t^7 + \\
& \frac{c_1^2 c_3 d_1^2 t^7}{2880} - \frac{c_1 c_4 d_1^2 t^7}{2880} - \frac{c_1^4 d_1^3 t^7}{4320} + \frac{c_1^2 c_2 d_1^3 t^7}{1080} + \frac{c_2^2 d_1^3 t^7}{1440} + \frac{c_1 c_3 d_1^3 t^7}{4320} - \\
& \frac{c_4 d_1^3 t^7}{4320} + \frac{1}{576} c_1 c_2 d_1^4 t^7 + \frac{c_1^2 d_1^5 t^7}{1440} + \frac{c_2 d_1^5 t^7}{1440} + \frac{c_1 d_1^6 t^7}{1440} + \frac{c_1^3 c_2 d_2 t^7}{1440} - \\
& \frac{1}{480} c_1 c_2^2 d_2 t^7 - \frac{c_1^2 c_3 d_2 t^7}{1440} + \frac{c_1 c_4 d_2 t^7}{1440} + \frac{c_1^4 d_1 d_2 t^7}{1440} - \frac{1}{360} c_1^2 c_2 d_1 d_2 t^7 - \\
& \frac{1}{480} c_2^2 d_1 d_2 t^7 - \frac{c_1 c_3 d_1 d_2 t^7}{1440} + \frac{c_4 d_1 d_2 t^7}{1440} - \frac{1}{144} c_1 c_2 d_1^2 d_2 t^7 - \frac{1}{288} c_1^2 d_1^3 d_2 t^7 - \\
& \frac{1}{288} c_2 d_1^3 d_2 t^7 - \frac{1}{240} c_1 d_1^4 d_2 t^7 + \frac{1}{288} c_1 c_2 d_2^2 t^7 + \frac{1}{288} c_1^2 d_1 d_2^2 t^7 + \\
& \frac{1}{288} c_2 d_1 d_2^2 t^7 + \frac{1}{160} c_1 d_1^2 d_2^2 t^7 - \frac{1}{720} c_1 d_2^3 t^7 - \frac{c_1^4 d_3 t^7}{1440} + \frac{1}{360} c_1^2 c_2 d_3 t^7 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{480} c_2^2 d_3 t^7 + \frac{c_1 c_3 d_3 t^7}{1440} - \frac{c_4 d_3 t^7}{1440} + \frac{1}{144} c_1 c_2 d_1 d_3 t^7 + \frac{1}{288} c_1^2 d_1^2 d_3 t^7 + \\
& \frac{1}{288} c_2 d_1^2 d_3 t^7 + \frac{1}{240} c_1 d_1^3 d_3 t^7 - \frac{1}{288} c_1^2 d_2 d_3 t^7 - \frac{1}{288} c_2 d_2 d_3 t^7 - \\
& \frac{1}{120} c_1 d_1 d_2 d_3 t^7 + \frac{1}{480} c_1 d_3^2 t^7 - \frac{1}{144} c_1 c_2 d_4 t^7 - \frac{1}{288} c_1^2 d_1 d_4 t^7 - \frac{1}{288} c_2 d_1 d_4 t^7 - \\
& \frac{1}{240} c_1 d_1^2 d_4 t^7 + \frac{1}{240} c_1 d_2 d_4 t^7 + \frac{1}{288} c_1^2 d_5 t^7 + \frac{1}{288} c_2 d_5 t^7 + \frac{1}{240} c_1 d_1 d_5 t^7 - \\
& \frac{1}{240} c_1 d_6 t^7 + \frac{c_1^6 d_1^2 t^8}{60480} - \frac{c_1^4 c_2 d_1^2 t^8}{10080} + \frac{11 c_1^2 c_2^2 d_1^2 t^8}{120960} + \frac{c_2^3 d_1^2 t^8}{12096} + \\
& \frac{c_1^3 c_3 d_1^2 t^8}{24192} + \frac{11 c_1 c_2 c_3 d_1^2 t^8}{120960} - \frac{c_3^2 d_1^2 t^8}{120960} - \frac{c_1^2 c_4 d_1^2 t^8}{24192} - \frac{c_2 c_4 d_1^2 t^8}{13440} - \\
& \frac{c_1 c_5 d_1^2 t^8}{60480} + \frac{c_6 d_1^2 t^8}{60480} - \frac{c_1^3 c_2 d_1^3 t^8}{8640} + \frac{c_1 c_2^2 d_1^3 t^8}{2880} + \frac{c_1^2 c_3 d_1^3 t^8}{8640} - \frac{c_1 c_4 d_1^3 t^8}{8640} - \\
& \frac{c_1^4 d_1^4 t^8}{17280} + \frac{c_1^2 c_2 d_1^4 t^8}{4320} + \frac{c_2^2 d_1^4 t^8}{5760} + \frac{c_1 c_3 d_1^4 t^8}{17280} - \frac{c_4 d_1^4 t^8}{17280} + \frac{c_1 c_2 d_1^5 t^8}{2880} + \\
& \frac{c_1^2 d_1^6 t^8}{8640} + \frac{c_2 d_1^6 t^8}{8640} - \frac{c_1^6 d_2 t^8}{30240} + \frac{c_1^4 c_2 d_2 t^8}{5040} - \frac{11 c_1^2 c_2^2 d_2 t^8}{60480} - \frac{c_2^3 d_2 t^8}{6048} - \\
& \frac{c_1^3 c_3 d_2 t^8}{12096} - \frac{11 c_1 c_2 c_3 d_2 t^8}{60480} + \frac{c_3^2 d_2 t^8}{60480} + \frac{c_1^2 c_4 d_2 t^8}{12096} + \frac{c_2 c_4 d_2 t^8}{6720} + \frac{c_1 c_5 d_2 t^8}{30240} - \\
& \frac{c_6 d_2 t^8}{30240} + \frac{c_1^3 c_2 d_1 d_2 t^8}{2880} - \frac{1}{960} c_1 c_2^2 d_1 d_2 t^8 - \frac{c_1^2 c_3 d_1 d_2 t^8}{2880} + \frac{c_1 c_4 d_1 d_2 t^8}{2880} + \\
& \frac{c_1^4 d_1^2 d_2 t^8}{4320} - \frac{c_1^2 c_2 d_1^2 d_2 t^8}{1080} - \frac{c_2^2 d_1^2 d_2 t^8}{1440} - \frac{c_1 c_3 d_1^2 d_2 t^8}{4320} + \frac{c_4 d_1^2 d_2 t^8}{4320} - \\
& \frac{1}{576} c_1 c_2 d_1^3 d_2 t^8 - \frac{c_1^2 d_1^4 d_2 t^8}{1440} - \frac{c_2 d_1^4 d_2 t^8}{1440} - \frac{c_1^4 d_2^2 t^8}{8640} + \frac{c_1^2 c_2 d_2^2 t^8}{2160} + \\
& \frac{c_2^2 d_2^2 t^8}{2880} + \frac{c_1 c_3 d_2^2 t^8}{8640} - \frac{c_4 d_2^2 t^8}{8640} + \frac{1}{576} c_1 c_2 d_1 d_2^2 t^8 + \frac{1}{960} c_1^2 d_1^2 d_2^2 t^8 + \\
& \frac{1}{960} c_2 d_1^2 d_2^2 t^8 - \frac{c_1^2 d_2^3 t^8}{4320} - \frac{c_2 d_2^3 t^8}{4320} - \frac{c_1^3 c_2 d_3 t^8}{2880} + \frac{1}{960} c_1 c_2^2 d_3 t^8 + \\
& \frac{c_1^2 c_3 d_3 t^8}{2880} - \frac{c_1 c_4 d_3 t^8}{2880} - \frac{c_1^4 d_1 d_3 t^8}{4320} + \frac{c_1^2 c_2 d_1 d_3 t^8}{1080} + \frac{c_2^2 d_1 d_3 t^8}{1440} + \\
& \frac{c_1 c_3 d_1 d_3 t^8}{4320} - \frac{c_4 d_1 d_3 t^8}{4320} + \frac{1}{576} c_1 c_2 d_1^2 d_3 t^8 + \frac{c_1^2 d_1^3 d_3 t^8}{1440} + \frac{c_2 d_1^3 d_3 t^8}{1440} - \\
& \frac{1}{576} c_1 c_2 d_2 d_3 t^8 - \frac{1}{720} c_1^2 d_1 d_2 d_3 t^8 - \frac{1}{720} c_2 d_1 d_2 d_3 t^8 + \frac{c_1^2 d_3^2 t^8}{2880} + \frac{c_2 d_3^2 t^8}{2880} + \\
& \frac{c_1^4 d_4 t^8}{4320} - \frac{c_1^2 c_2 d_4 t^8}{1080} - \frac{c_2^2 d_4 t^8}{1440} - \frac{c_1 c_3 d_4 t^8}{4320} + \frac{c_4 d_4 t^8}{4320} - \frac{1}{576} c_1 c_2 d_1 d_4 t^8 - \\
& \frac{c_1^2 d_1^2 d_4 t^8}{1440} - \frac{c_2 d_1^2 d_4 t^8}{1440} + \frac{c_1^2 d_2 d_4 t^8}{1440} + \frac{c_2 d_2 d_4 t^8}{1440} + \frac{1}{576} c_1 c_2 d_5 t^8 + \\
& \frac{c_1^2 d_1 d_5 t^8}{1440} + \frac{c_2 d_1 d_5 t^8}{1440} - \frac{c_1^2 d_6 t^8}{1440} - \frac{c_2 d_6 t^8}{1440} + \frac{c_1^6 d_1^3 t^9}{181440} - \frac{c_1^4 c_2 d_1^3 t^9}{30240} + \\
& \frac{11 c_1^2 c_2^2 d_1^3 t^9}{362880} + \frac{c_2^3 d_1^3 t^9}{36288} + \frac{c_1^3 c_3 d_1^3 t^9}{72576} + \frac{11 c_1 c_2 c_3 d_1^3 t^9}{362880} - \frac{c_3^2 d_1^3 t^9}{362880} -
\end{aligned}$$

$$\begin{aligned}
 & \frac{c1^2 c4 d1^3 t^9}{72576} - \frac{c2 c4 d1^3 t^9}{40320} - \frac{c1 c5 d1^3 t^9}{181440} + \frac{c6 d1^3 t^9}{181440} - \frac{c1^3 c2 d1^4 t^9}{34560} + \frac{c1 c2^2 d1^4 t^9}{11520} + \\
 & \frac{c1^2 c3 d1^4 t^9}{34560} - \frac{c1 c4 d1^4 t^9}{34560} - \frac{c1^4 d1^5 t^9}{86400} + \frac{c1^2 c2 d1^5 t^9}{21600} + \frac{c2^2 d1^5 t^9}{28800} + \frac{c1 c3 d1^5 t^9}{86400} - \\
 & \frac{c4 d1^5 t^9}{86400} + \frac{c1 c2 d1^6 t^9}{17280} - \frac{c1^6 d1 d2 t^9}{60480} + \frac{c1^4 c2 d1 d2 t^9}{10080} - \frac{11 c1^2 c2^2 d1 d2 t^9}{120960} - \\
 & \frac{c2^3 d1 d2 t^9}{12096} - \frac{c1^3 c3 d1 d2 t^9}{24192} - \frac{11 c1 c2 c3 d1 d2 t^9}{120960} + \frac{c3^2 d1 d2 t^9}{120960} + \frac{c1^2 c4 d1 d2 t^9}{24192} + \\
 & \frac{c2 c4 d1 d2 t^9}{13440} + \frac{c1 c5 d1 d2 t^9}{60480} - \frac{c6 d1 d2 t^9}{60480} + \frac{c1^3 c2 d1^2 d2 t^9}{8640} - \frac{c1 c2^2 d1^2 d2 t^9}{2880} - \\
 & \frac{c1^2 c3 d1^2 d2 t^9}{8640} + \frac{c1 c4 d1^2 d2 t^9}{8640} + \frac{c1^4 d1^3 d2 t^9}{17280} - \frac{c1^2 c2 d1^3 d2 t^9}{4320} - \frac{c2^2 d1^3 d2 t^9}{5760} - \\
 & \frac{c1 c3 d1^3 d2 t^9}{17280} + \frac{c4 d1^3 d2 t^9}{17280} - \frac{c1 c2 d1^4 d2 t^9}{2880} - \frac{c1^3 c2 d2^2 t^9}{17280} + \frac{c1 c2^2 d2^2 t^9}{5760} + \\
 & \frac{c1^2 c3 d2^2 t^9}{17280} - \frac{c1 c4 d2^2 t^9}{17280} - \frac{c1^4 d1 d2^2 t^9}{17280} + \frac{c1^2 c2 d1 d2^2 t^9}{4320} + \frac{c2^2 d1 d2^2 t^9}{5760} + \\
 & \frac{c1 c3 d1 d2^2 t^9}{17280} - \frac{c4 d1 d2^2 t^9}{17280} + \frac{c1 c2 d1^2 d2^2 t^9}{1920} - \frac{c1 c2 d2^3 t^9}{8640} + \frac{c1^6 d3 t^9}{60480} - \\
 & \frac{c1^4 c2 d3 t^9}{10080} + \frac{11 c1^2 c2^2 d3 t^9}{120960} + \frac{c2^3 d3 t^9}{12096} + \frac{c1^3 c3 d3 t^9}{24192} + \frac{11 c1 c2 c3 d3 t^9}{120960} - \\
 & \frac{c3^2 d3 t^9}{120960} - \frac{c1^2 c4 d3 t^9}{24192} - \frac{c2 c4 d3 t^9}{13440} - \frac{c1 c5 d3 t^9}{60480} + \frac{c6 d3 t^9}{60480} - \frac{c1^3 c2 d1 d3 t^9}{8640} + \\
 & \frac{c1 c2^2 d1 d3 t^9}{2880} + \frac{c1^2 c3 d1 d3 t^9}{8640} - \frac{c1 c4 d1 d3 t^9}{8640} - \frac{c1^4 d1^2 d3 t^9}{17280} + \frac{c1^2 c2 d1^2 d3 t^9}{4320} + \\
 & \frac{c2^2 d1^2 d3 t^9}{5760} + \frac{c1 c3 d1^2 d3 t^9}{17280} - \frac{c4 d1^2 d3 t^9}{17280} + \frac{c1 c2 d1^3 d3 t^9}{2880} + \frac{c1^4 d2 d3 t^9}{17280} - \\
 & \frac{c1^2 c2 d2 d3 t^9}{4320} - \frac{c2^2 d2 d3 t^9}{5760} - \frac{c1 c3 d2 d3 t^9}{17280} + \frac{c4 d2 d3 t^9}{17280} - \frac{c1 c2 d1 d2 d3 t^9}{1440} + \\
 & \frac{c1 c2 d3^2 t^9}{5760} + \frac{c1^3 c2 d4 t^9}{8640} - \frac{c1 c2^2 d4 t^9}{2880} - \frac{c1^2 c3 d4 t^9}{8640} + \frac{c1 c4 d4 t^9}{8640} + \frac{c1^4 d1 d4 t^9}{17280} - \\
 & \frac{c1^2 c2 d1 d4 t^9}{4320} - \frac{c2^2 d1 d4 t^9}{5760} - \frac{c1 c3 d1 d4 t^9}{17280} + \frac{c4 d1 d4 t^9}{17280} - \frac{c1 c2 d1^2 d4 t^9}{2880} + \\
 & \frac{c1 c2 d2 d4 t^9}{2880} - \frac{c1^4 d5 t^9}{17280} + \frac{c1^2 c2 d5 t^9}{4320} + \frac{c2^2 d5 t^9}{5760} + \frac{c1 c3 d5 t^9}{17280} - \frac{c4 d5 t^9}{17280} + \\
 & \frac{c1 c2 d1 d5 t^9}{2880} - \frac{c1 c2 d6 t^9}{2880} + \frac{c1^6 d1^4 t^{10}}{725760} - \frac{c1^4 c2 d1^4 t^{10}}{120960} + \frac{11 c1^2 c2^2 d1^4 t^{10}}{1451520} + \\
 & \frac{c2^3 d1^4 t^{10}}{145152} + \frac{c1^3 c3 d1^4 t^{10}}{290304} + \frac{11 c1 c2 c3 d1^4 t^{10}}{1451520} - \frac{c3^2 d1^4 t^{10}}{1451520} - \frac{c1^2 c4 d1^4 t^{10}}{290304} - \\
 & \frac{c2 c4 d1^4 t^{10}}{161280} - \frac{c1 c5 d1^4 t^{10}}{725760} + \frac{c6 d1^4 t^{10}}{725760} - \frac{c1^3 c2 d1^5 t^{10}}{172800} + \frac{c1 c2^2 d1^5 t^{10}}{57600} + \\
 & \frac{c1^2 c3 d1^5 t^{10}}{172800} - \frac{c1 c4 d1^5 t^{10}}{172800} - \frac{c1^4 d1^6 t^{10}}{518400} + \frac{c1^2 c2 d1^6 t^{10}}{129600} + \frac{c2^2 d1^6 t^{10}}{172800} + \frac{c1 c3 d1^6 t^{10}}{518400} - \\
 & \frac{c4 d1^6 t^{10}}{518400} - \frac{c1^6 d1^2 d2 t^{10}}{181440} + \frac{c1^4 c2 d1^2 d2 t^{10}}{30240} - \frac{11 c1^2 c2^2 d1^2 d2 t^{10}}{362880} - \frac{c2^3 d1^2 d2 t^{10}}{36288} -
 \end{aligned}$$

$$\begin{aligned}
& \frac{c1^3 c3 d1^2 d2 t^{10}}{72576} - \frac{11 c1 c2 c3 d1^2 d2 t^{10}}{362880} + \frac{c3^2 d1^2 d2 t^{10}}{362880} + \frac{c1^2 c4 d1^2 d2 t^{10}}{72576} + \\
& \frac{c2 c4 d1^2 d2 t^{10}}{40320} + \frac{c1 c5 d1^2 d2 t^{10}}{181440} - \frac{c6 d1^2 d2 t^{10}}{181440} + \frac{c1^3 c2 d1^3 d2 t^{10}}{34560} - \frac{c1 c2^2 d1^3 d2 t^{10}}{11520} - \\
& \frac{c1^2 c3 d1^3 d2 t^{10}}{34560} + \frac{c1 c4 d1^3 d2 t^{10}}{34560} + \frac{c1^4 d1^4 d2 t^{10}}{86400} - \frac{c1^2 c2 d1^4 d2 t^{10}}{21600} - \frac{c2^2 d1^4 d2 t^{10}}{28800} - \\
& \frac{c1 c3 d1^4 d2 t^{10}}{86400} + \frac{c4 d1^4 d2 t^{10}}{86400} + \frac{c1^6 d2^2 t^{10}}{362880} - \frac{c1^4 c2 d2^2 t^{10}}{60480} + \frac{11 c1^2 c2^2 d2^2 t^{10}}{725760} + \\
& \frac{c2^3 d2^2 t^{10}}{72576} + \frac{c1^3 c3 d2^2 t^{10}}{145152} + \frac{11 c1 c2 c3 d2^2 t^{10}}{725760} - \frac{c3^2 d2^2 t^{10}}{725760} - \frac{c1^2 c4 d2^2 t^{10}}{145152} - \\
& \frac{c2 c4 d2^2 t^{10}}{80640} - \frac{c1 c5 d2^2 t^{10}}{362880} + \frac{c6 d2^2 t^{10}}{362880} - \frac{c1^3 c2 d1 d2^2 t^{10}}{34560} + \frac{c1 c2^2 d1 d2^2 t^{10}}{11520} + \\
& \frac{c1^2 c3 d1 d2^2 t^{10}}{34560} - \frac{c1 c4 d1 d2^2 t^{10}}{34560} - \frac{c1^4 d1^2 d2^2 t^{10}}{57600} + \frac{c1^2 c2 d1^2 d2^2 t^{10}}{14400} + \\
& \frac{c2^2 d1^2 d2^2 t^{10}}{19200} + \frac{c1 c3 d1^2 d2^2 t^{10}}{57600} - \frac{c4 d1^2 d2^2 t^{10}}{57600} + \frac{c1^4 d2^3 t^{10}}{259200} - \frac{c1^2 c2 d2^3 t^{10}}{64800} - \\
& \frac{c2^2 d2^3 t^{10}}{86400} - \frac{c1 c3 d2^3 t^{10}}{259200} + \frac{c4 d2^3 t^{10}}{259200} + \frac{c1^6 d1 d3 t^{10}}{181440} - \frac{c1^4 c2 d1 d3 t^{10}}{30240} + \\
& \frac{11 c1^2 c2^2 d1 d3 t^{10}}{362880} + \frac{c2^3 d1 d3 t^{10}}{36288} + \frac{c1^3 c3 d1 d3 t^{10}}{72576} + \frac{11 c1 c2 c3 d1 d3 t^{10}}{362880} - \\
& \frac{c3^2 d1 d3 t^{10}}{362880} - \frac{c1^2 c4 d1 d3 t^{10}}{72576} - \frac{c2 c4 d1 d3 t^{10}}{40320} - \frac{c1 c5 d1 d3 t^{10}}{181440} + \frac{c6 d1 d3 t^{10}}{181440} - \\
& \frac{c1^3 c2 d1^2 d3 t^{10}}{34560} + \frac{c1 c2^2 d1^2 d3 t^{10}}{11520} + \frac{c1^2 c3 d1^2 d3 t^{10}}{34560} - \frac{c1 c4 d1^2 d3 t^{10}}{34560} - \\
& \frac{c1^4 d1^3 d3 t^{10}}{86400} + \frac{c1^2 c2 d1^3 d3 t^{10}}{21600} + \frac{c2^2 d1^3 d3 t^{10}}{28800} + \frac{c1 c3 d1^3 d3 t^{10}}{86400} - \frac{c4 d1^3 d3 t^{10}}{86400} + \\
& \frac{c1^3 c2 d2 d3 t^{10}}{34560} - \frac{c1 c2^2 d2 d3 t^{10}}{11520} - \frac{c1^2 c3 d2 d3 t^{10}}{34560} + \frac{c1 c4 d2 d3 t^{10}}{34560} + \frac{c1^4 d1 d2 d3 t^{10}}{43200} - \\
& \frac{c1^2 c2 d1 d2 d3 t^{10}}{10800} - \frac{c2^2 d1 d2 d3 t^{10}}{14400} - \frac{c1 c3 d1 d2 d3 t^{10}}{43200} + \frac{c4 d1 d2 d3 t^{10}}{43200} - \\
& \frac{c1^4 d3^2 t^{10}}{172800} + \frac{c1^2 c2 d3^2 t^{10}}{43200} + \frac{c2^2 d3^2 t^{10}}{57600} + \frac{c1 c3 d3^2 t^{10}}{172800} - \frac{c4 d3^2 t^{10}}{172800} - \frac{c1^6 d4 t^{10}}{181440} + \\
& \frac{c1^4 c2 d4 t^{10}}{30240} - \frac{11 c1^2 c2^2 d4 t^{10}}{362880} - \frac{c2^3 d4 t^{10}}{36288} - \frac{c1^3 c3 d4 t^{10}}{72576} - \frac{11 c1 c2 c3 d4 t^{10}}{362880} + \\
& \frac{c3^2 d4 t^{10}}{362880} + \frac{c1^2 c4 d4 t^{10}}{72576} + \frac{c2 c4 d4 t^{10}}{40320} + \frac{c1 c5 d4 t^{10}}{181440} - \frac{c6 d4 t^{10}}{181440} + \frac{c1^3 c2 d1 d4 t^{10}}{34560} - \\
& \frac{c1 c2^2 d1 d4 t^{10}}{11520} - \frac{c1^2 c3 d1 d4 t^{10}}{34560} + \frac{c1 c4 d1 d4 t^{10}}{34560} + \frac{c1^4 d1^2 d4 t^{10}}{86400} - \frac{c1^2 c2 d1^2 d4 t^{10}}{21600} - \\
& \frac{c2^2 d1^2 d4 t^{10}}{28800} - \frac{c1 c3 d1^2 d4 t^{10}}{86400} + \frac{c4 d1^2 d4 t^{10}}{86400} - \frac{c1^4 d2 d4 t^{10}}{86400} + \frac{c1^2 c2 d2 d4 t^{10}}{21600} + \\
& \frac{c2^2 d2 d4 t^{10}}{28800} + \frac{c1 c3 d2 d4 t^{10}}{86400} - \frac{c4 d2 d4 t^{10}}{86400} - \frac{c1^3 c2 d5 t^{10}}{34560} + \frac{c1 c2^2 d5 t^{10}}{11520} + \\
& \frac{c1^2 c3 d5 t^{10}}{34560} - \frac{c1 c4 d5 t^{10}}{34560} - \frac{c1^4 d1 d5 t^{10}}{86400} + \frac{c1^2 c2 d1 d5 t^{10}}{21600} + \frac{c2^2 d1 d5 t^{10}}{28800} + \\
& \frac{34560}{34560} - \frac{34560}{34560} - \frac{86400}{86400} + \frac{21600}{21600} + \frac{28800}{28800}
\end{aligned}$$

$$\begin{aligned}
 & \frac{c_1 c_3 d_1 d_5 t^{10}}{86400} - \frac{c_4 d_1 d_5 t^{10}}{86400} + \frac{c_1^4 d_6 t^{10}}{86400} - \frac{c_1^2 c_2 d_6 t^{10}}{21600} - \frac{c_2^2 d_6 t^{10}}{28800} - \frac{c_1 c_3 d_6 t^{10}}{86400} + \\
 & \frac{c_4 d_6 t^{10}}{86400} + \frac{c_1^6 d_1^5 t^{11}}{3628800} - \frac{c_1^4 c_2 d_1^5 t^{11}}{604800} + \frac{11 c_1^2 c_2^2 d_1^5 t^{11}}{7257600} - \frac{c_2^3 d_1^5 t^{11}}{725760} + \\
 & \frac{c_1^3 c_3 d_1^5 t^{11}}{1451520} + \frac{11 c_1 c_2 c_3 d_1^5 t^{11}}{7257600} - \frac{c_3^2 d_1^5 t^{11}}{7257600} - \frac{c_1^2 c_4 d_1^5 t^{11}}{1451520} - \frac{c_2 c_4 d_1^5 t^{11}}{806400} - \\
 & \frac{c_1 c_5 d_1^5 t^{11}}{3628800} + \frac{c_6 d_1^5 t^{11}}{3628800} - \frac{c_1^3 c_2 d_1^6 t^{11}}{1036800} + \frac{c_1 c_2^2 d_1^6 t^{11}}{345600} + \frac{c_1^2 c_3 d_1^6 t^{11}}{1036800} - \\
 & \frac{c_1 c_4 d_1^6 t^{11}}{1036800} - \frac{c_1^6 d_1^3 d_2 t^{11}}{725760} + \frac{c_1^4 c_2 d_1^3 d_2 t^{11}}{120960} - \frac{11 c_1^2 c_2^2 d_1^3 d_2 t^{11}}{1451520} - \frac{c_2^3 d_1^3 d_2 t^{11}}{145152} - \\
 & \frac{c_1^3 c_3 d_1^3 d_2 t^{11}}{290304} - \frac{11 c_1 c_2 c_3 d_1^3 d_2 t^{11}}{1451520} + \frac{c_3^2 d_1^3 d_2 t^{11}}{1451520} + \frac{c_1^2 c_4 d_1^3 d_2 t^{11}}{290304} + \\
 & \frac{c_2 c_4 d_1^3 d_2 t^{11}}{161280} + \frac{c_1 c_5 d_1^3 d_2 t^{11}}{725760} - \frac{c_6 d_1^3 d_2 t^{11}}{725760} + \frac{c_1^3 c_2 d_1^4 d_2 t^{11}}{172800} - \frac{c_1 c_2^2 d_1^4 d_2 t^{11}}{57600} - \\
 & \frac{c_1^2 c_3 d_1^4 d_2 t^{11}}{172800} + \frac{c_1 c_4 d_1^4 d_2 t^{11}}{172800} + \frac{c_1^6 d_1 d_2^2 t^{11}}{725760} - \frac{c_1^4 c_2 d_1 d_2^2 t^{11}}{120960} + \\
 & \frac{11 c_1^2 c_2^2 d_1 d_2^2 t^{11}}{1451520} + \frac{c_2^3 d_1 d_2^2 t^{11}}{145152} + \frac{c_1^3 c_3 d_1 d_2^2 t^{11}}{290304} + \frac{11 c_1 c_2 c_3 d_1 d_2^2 t^{11}}{1451520} - \\
 & \frac{c_3^2 d_1 d_2^2 t^{11}}{1451520} - \frac{c_1^2 c_4 d_1 d_2^2 t^{11}}{290304} - \frac{c_2 c_4 d_1 d_2^2 t^{11}}{161280} - \frac{c_1 c_5 d_1 d_2^2 t^{11}}{725760} + \frac{c_6 d_1 d_2^2 t^{11}}{725760} - \\
 & \frac{c_1^3 c_2 d_1^2 d_2^2 t^{11}}{115200} + \frac{c_1 c_2^2 d_1^2 d_2^2 t^{11}}{38400} + \frac{c_1^2 c_3 d_1^2 d_2^2 t^{11}}{115200} - \frac{c_1 c_4 d_1^2 d_2^2 t^{11}}{115200} + \\
 & \frac{c_1^3 c_2 d_2^3 t^{11}}{518400} - \frac{c_1 c_2^2 d_2^3 t^{11}}{172800} - \frac{c_1^2 c_3 d_2^3 t^{11}}{518400} + \frac{c_1 c_4 d_2^3 t^{11}}{518400} + \frac{c_1^6 d_1^2 d_3 t^{11}}{725760} - \\
 & \frac{c_1^4 c_2 d_1^2 d_3 t^{11}}{120960} + \frac{11 c_1^2 c_2^2 d_1^2 d_3 t^{11}}{1451520} + \frac{c_2^3 d_1^2 d_3 t^{11}}{145152} + \frac{c_1^3 c_3 d_1^2 d_3 t^{11}}{290304} + \\
 & \frac{11 c_1 c_2 c_3 d_1^2 d_3 t^{11}}{1451520} - \frac{c_3^2 d_1^2 d_3 t^{11}}{1451520} - \frac{c_1^2 c_4 d_1^2 d_3 t^{11}}{290304} - \frac{c_2 c_4 d_1^2 d_3 t^{11}}{161280} - \\
 & \frac{c_1 c_5 d_1^2 d_3 t^{11}}{725760} + \frac{c_6 d_1^2 d_3 t^{11}}{725760} - \frac{c_1^2 c_3 d_1^3 d_3 t^{11}}{172800} + \frac{c_1 c_2^2 d_1^3 d_3 t^{11}}{57600} + \frac{c_1^2 c_3 d_1^3 d_3 t^{11}}{172800} - \\
 & \frac{c_1 c_4 d_1^3 d_3 t^{11}}{172800} - \frac{c_1^6 d_2 d_3 t^{11}}{725760} + \frac{c_1^4 c_2 d_2 d_3 t^{11}}{120960} - \frac{11 c_1^2 c_2^2 d_2 d_3 t^{11}}{1451520} - \\
 & \frac{c_2^3 d_2 d_3 t^{11}}{1451520} + \frac{c_1^3 c_3 d_2 d_3 t^{11}}{1451520} + \\
 & \frac{c_1^2 c_4 d_2 d_3 t^{11}}{290304} + \frac{c_2 c_4 d_2 d_3 t^{11}}{161280} + \frac{c_1 c_5 d_2 d_3 t^{11}}{725760} - \frac{c_6 d_2 d_3 t^{11}}{725760} + \frac{c_1^3 c_2 d_1 d_2 d_3 t^{11}}{86400} - \\
 & \frac{c_1 c_2^2 d_1 d_2 d_3 t^{11}}{28800} - \frac{c_1^2 c_3 d_1 d_2 d_3 t^{11}}{86400} + \frac{c_1 c_4 d_1 d_2 d_3 t^{11}}{86400} - \frac{c_1^3 c_2 d_3^2 t^{11}}{345600} + \\
 & \frac{c_1 c_2^2 d_3^2 t^{11}}{115200} + \frac{c_1^2 c_3 d_3^2 t^{11}}{345600} - \frac{c_1 c_4 d_3^2 t^{11}}{345600} - \frac{c_1^6 d_1 d_4 t^{11}}{725760} + \frac{c_1^4 c_2 d_1 d_4 t^{11}}{120960} - \\
 & \frac{11 c_1^2 c_2^2 d_1 d_4 t^{11}}{1451520} - \frac{c_2^3 d_1 d_4 t^{11}}{145152} - \frac{c_1^3 c_3 d_1 d_4 t^{11}}{290304} - \frac{11 c_1 c_2 c_3 d_1 d_4 t^{11}}{1451520} + \\
 & \frac{c_3^2 d_1 d_4 t^{11}}{1451520} + \frac{c_1^2 c_4 d_1 d_4 t^{11}}{145152} + \frac{c_2 c_4 d_1 d_4 t^{11}}{290304} + \frac{c_1 c_5 d_1 d_4 t^{11}}{725760} - \frac{c_6 d_1 d_4 t^{11}}{725760} + \\
 & \frac{1451520}{1451520} + \frac{290304}{290304} + \frac{161280}{161280} + \frac{725760}{725760} - \frac{725760}{725760} +
 \end{aligned}$$

$$\begin{aligned}
& \frac{c_1^3 c_2 d_1^2 d_4 t^{11}}{172\,800} - \frac{c_1 c_2^2 d_1^2 d_4 t^{11}}{57\,600} - \frac{c_1^2 c_3 d_1^2 d_4 t^{11}}{172\,800} + \frac{c_1 c_4 d_1^2 d_4 t^{11}}{172\,800} - \\
& \frac{c_1^3 c_2 d_2 d_4 t^{11}}{172\,800} + \frac{c_1 c_2^2 d_2 d_4 t^{11}}{57\,600} + \frac{c_1^2 c_3 d_2 d_4 t^{11}}{172\,800} - \frac{c_1 c_4 d_2 d_4 t^{11}}{172\,800} + \frac{c_1^6 d_5 t^{11}}{725\,760} - \\
& \frac{c_1^4 c_2 d_5 t^{11}}{120\,960} + \frac{11 c_1^2 c_2^2 d_5 t^{11}}{1\,451\,520} + \frac{c_2^3 d_5 t^{11}}{145\,152} + \frac{c_1^3 c_3 d_5 t^{11}}{290\,304} + \frac{11 c_1 c_2 c_3 d_5 t^{11}}{1\,451\,520} - \\
& \frac{c_3^2 d_5 t^{11}}{1\,451\,520} - \frac{c_1^2 c_4 d_5 t^{11}}{290\,304} - \frac{c_2 c_4 d_5 t^{11}}{161\,280} - \frac{c_1 c_5 d_5 t^{11}}{725\,760} + \frac{c_6 d_5 t^{11}}{725\,760} - \frac{c_1^3 c_2 d_1 d_5 t^{11}}{172\,800} + \\
& \frac{c_1 c_2^2 d_1 d_5 t^{11}}{57\,600} + \frac{c_1^2 c_3 d_1 d_5 t^{11}}{172\,800} - \frac{c_1 c_4 d_1 d_5 t^{11}}{172\,800} + \frac{c_1^3 c_2 d_6 t^{11}}{172\,800} - \frac{c_1 c_2^2 d_6 t^{11}}{57\,600} - \\
& \frac{c_1^2 c_3 d_6 t^{11}}{172\,800} + \frac{c_1 c_4 d_6 t^{11}}{172\,800} + \frac{c_1^6 d_1^6 t^{12}}{21\,772\,800} - \frac{c_1^4 c_2 d_1^6 t^{12}}{3\,628\,800} + \frac{11 c_1^2 c_2^2 d_1^6 t^{12}}{43\,545\,600} + \\
& \frac{c_2^3 d_1^6 t^{12}}{4\,354\,560} + \frac{c_1^3 c_3 d_1^6 t^{12}}{8\,709\,120} + \frac{11 c_1 c_2 c_3 d_1^6 t^{12}}{43\,545\,600} - \frac{c_3^2 d_1^6 t^{12}}{43\,545\,600} - \frac{c_1^2 c_4 d_1^6 t^{12}}{8\,709\,120} - \\
& \frac{c_2 c_4 d_1^6 t^{12}}{4\,838\,400} - \frac{c_1 c_5 d_1^6 t^{12}}{21\,772\,800} + \frac{c_6 d_1^6 t^{12}}{21\,772\,800} - \frac{c_1^6 d_1^4 d_2 t^{12}}{3\,628\,800} + \frac{c_1^4 c_2 d_1^4 d_2 t^{12}}{604\,800} - \\
& \frac{11 c_1^2 c_2^2 d_1^4 d_2 t^{12}}{7\,257\,600} - \frac{c_2^3 d_1^4 d_2 t^{12}}{725\,760} - \frac{c_1^3 c_3 d_1^4 d_2 t^{12}}{1\,451\,520} - \frac{11 c_1 c_2 c_3 d_1^4 d_2 t^{12}}{7\,257\,600} + \\
& \frac{c_3^2 d_1^4 d_2 t^{12}}{7\,257\,600} + \frac{c_1^2 c_4 d_1^4 d_2 t^{12}}{1\,451\,520} + \frac{c_2 c_4 d_1^4 d_2 t^{12}}{806\,400} + \frac{c_1 c_5 d_1^4 d_2 t^{12}}{3\,628\,800} - \\
& \frac{c_6 d_1^4 d_2 t^{12}}{3\,628\,800} + \frac{c_1^6 d_1^2 d_2^2 t^{12}}{2\,419\,200} - \frac{c_1^4 c_2 d_1^2 d_2^2 t^{12}}{403\,200} + \frac{11 c_1^2 c_2^2 d_1^2 d_2^2 t^{12}}{4\,838\,400} + \\
& \frac{c_2^3 d_1^2 d_2^2 t^{12}}{483\,840} + \frac{c_1^3 c_3 d_1^2 d_2^2 t^{12}}{967\,680} + \frac{11 c_1 c_2 c_3 d_1^2 d_2^2 t^{12}}{4\,838\,400} - \frac{c_3^2 d_1^2 d_2^2 t^{12}}{4\,838\,400} - \\
& \frac{c_1^2 c_4 d_1^2 d_2^2 t^{12}}{967\,680} - \frac{c_2 c_4 d_1^2 d_2^2 t^{12}}{537\,600} - \frac{c_1 c_5 d_1^2 d_2^2 t^{12}}{2\,419\,200} + \frac{c_6 d_1^2 d_2^2 t^{12}}{2\,419\,200} - \\
& \frac{c_1^6 d_2^3 t^{12}}{10\,886\,400} + \frac{c_1^4 c_2 d_2^3 t^{12}}{1\,814\,400} - \frac{11 c_1^2 c_2^2 d_2^3 t^{12}}{21\,772\,800} - \frac{c_2^3 d_2^3 t^{12}}{2\,177\,280} - \frac{c_1^3 c_3 d_2^3 t^{12}}{4\,354\,560} - \\
& \frac{11 c_1 c_2 c_3 d_2^3 t^{12}}{21\,772\,800} + \frac{c_3^2 d_2^3 t^{12}}{21\,772\,800} + \frac{c_1^2 c_4 d_2^3 t^{12}}{4\,354\,560} + \frac{c_2 c_4 d_2^3 t^{12}}{2\,419\,200} + \frac{c_1 c_5 d_2^3 t^{12}}{10\,886\,400} - \\
& \frac{c_6 d_2^3 t^{12}}{10\,886\,400} + \frac{c_1^6 d_1^3 d_3 t^{12}}{3\,628\,800} - \frac{c_1^4 c_2 d_1^3 d_3 t^{12}}{604\,800} + \frac{11 c_1^2 c_2^2 d_1^3 d_3 t^{12}}{7\,257\,600} + \frac{c_2^3 d_1^3 d_3 t^{12}}{725\,760} + \\
& \frac{c_1^3 c_3 d_1^3 d_3 t^{12}}{1\,451\,520} + \frac{11 c_1 c_2 c_3 d_1^3 d_3 t^{12}}{7\,257\,600} - \frac{c_3^2 d_1^3 d_3 t^{12}}{7\,257\,600} - \frac{c_1^2 c_4 d_1^3 d_3 t^{12}}{1\,451\,520} - \\
& \frac{c_2 c_4 d_1^3 d_3 t^{12}}{806\,400} - \frac{c_1 c_5 d_1^3 d_3 t^{12}}{3\,628\,800} + \frac{c_6 d_1^3 d_3 t^{12}}{3\,628\,800} - \frac{c_1^6 d_1 d_2 d_3 t^{12}}{1\,814\,400} + \\
& \frac{c_1^4 c_2 d_1 d_2 d_3 t^{12}}{302\,400} - \frac{11 c_1^2 c_2^2 d_1 d_2 d_3 t^{12}}{3\,628\,800} - \frac{c_2^3 d_1 d_2 d_3 t^{12}}{362\,880} - \frac{c_1^3 c_3 d_1 d_2 d_3 t^{12}}{725\,760} - \\
& \frac{11 c_1 c_2 c_3 d_1 d_2 d_3 t^{12}}{3\,628\,800} + \frac{c_3^2 d_1 d_2 d_3 t^{12}}{3\,628\,800} + \frac{c_1^2 c_4 d_1 d_2 d_3 t^{12}}{725\,760} + \frac{c_2 c_4 d_1 d_2 d_3 t^{12}}{403\,200} + \\
& \frac{c_1 c_5 d_1 d_2 d_3 t^{12}}{1\,814\,400} - \frac{c_6 d_1 d_2 d_3 t^{12}}{1\,814\,400} + \frac{c_1^6 d_3^2 t^{12}}{7\,257\,600} - \frac{c_1^4 c_2 d_3^2 t^{12}}{1\,209\,600} + \frac{11 c_1^2 c_2^2 d_3^2 t^{12}}{14\,515\,200} + \\
& \frac{c_2^3 d_3^2 t^{12}}{1\,451\,520} + \frac{c_1^3 c_3 d_3^2 t^{12}}{2\,903\,040} + \frac{11 c_1 c_2 c_3 d_3^2 t^{12}}{14\,515\,200} - \frac{c_3^2 d_3^2 t^{12}}{1\,451\,520} - \frac{c_1^2 c_4 d_3^2 t^{12}}{2\,903\,040} - \\
& \frac{1\,451\,520}{1\,451\,520} + \frac{2\,903\,040}{2\,903\,040} + \frac{14\,515\,200}{14\,515\,200} - \frac{1\,451\,520}{1\,451\,520} - \frac{2\,903\,040}{2\,903\,040} -
\end{aligned}$$

$$\begin{aligned}
& \frac{c_2 c_4 d_3^2 t^{12}}{1612800} - \frac{c_1 c_5 d_3^2 t^{12}}{7257600} + \frac{c_6 d_3^2 t^{12}}{7257600} - \frac{c_1^6 d_1^2 d_4 t^{12}}{3628800} + \frac{c_1^4 c_2 d_1^2 d_4 t^{12}}{604800} - \\
& \frac{11 c_1^2 c_2^2 d_1^2 d_4 t^{12}}{7257600} - \frac{c_2^3 d_1^2 d_4 t^{12}}{725760} - \frac{c_1^3 c_3 d_1^2 d_4 t^{12}}{1451520} - \frac{11 c_1 c_2 c_3 d_1^2 d_4 t^{12}}{7257600} + \\
& \frac{c_3^2 d_1^2 d_4 t^{12}}{7257600} + \frac{c_1^2 c_4 d_1^2 d_4 t^{12}}{1451520} + \frac{c_2 c_4 d_1^2 d_4 t^{12}}{806400} + \frac{c_1 c_5 d_1^2 d_4 t^{12}}{3628800} - \\
& \frac{c_6 d_1^2 d_4 t^{12}}{3628800} + \frac{c_1^6 d_2 d_4 t^{12}}{3628800} - \frac{c_1^4 c_2 d_2 d_4 t^{12}}{604800} + \frac{11 c_1^2 c_2^2 d_2 d_4 t^{12}}{7257600} + \frac{c_2^3 d_2 d_4 t^{12}}{725760} + \\
& \frac{c_1^3 c_3 d_2 d_4 t^{12}}{1451520} + \frac{11 c_1 c_2 c_3 d_2 d_4 t^{12}}{7257600} - \frac{c_3^2 d_2 d_4 t^{12}}{7257600} - \frac{c_1^2 c_4 d_2 d_4 t^{12}}{1451520} - \\
& \frac{c_2 c_4 d_2 d_4 t^{12}}{806400} - \frac{c_1 c_5 d_2 d_4 t^{12}}{3628800} + \frac{c_6 d_2 d_4 t^{12}}{3628800} + \frac{c_1^6 d_1 d_5 t^{12}}{3628800} - \frac{c_1^4 c_2 d_1 d_5 t^{12}}{604800} + \\
& \frac{11 c_1^2 c_2^2 d_1 d_5 t^{12}}{7257600} + \frac{c_2^3 d_1 d_5 t^{12}}{725760} + \frac{c_1^3 c_3 d_1 d_5 t^{12}}{1451520} + \frac{11 c_1 c_2 c_3 d_1 d_5 t^{12}}{7257600} - \\
& \frac{c_3^2 d_1 d_5 t^{12}}{7257600} - \frac{c_1^2 c_4 d_1 d_5 t^{12}}{1451520} - \frac{c_2 c_4 d_1 d_5 t^{12}}{806400} - \frac{c_1 c_5 d_1 d_5 t^{12}}{3628800} + \frac{c_6 d_1 d_5 t^{12}}{3628800} - \\
& \frac{c_1^6 d_6 t^{12}}{3628800} + \frac{c_1^4 c_2 d_6 t^{12}}{604800} - \frac{11 c_1^2 c_2^2 d_6 t^{12}}{7257600} - \frac{c_2^3 d_6 t^{12}}{725760} - \frac{c_1^3 c_3 d_6 t^{12}}{1451520} - \\
& \frac{11 c_1 c_2 c_3 d_6 t^{12}}{7257600} + \frac{c_3^2 d_6 t^{12}}{7257600} + \frac{c_1^2 c_4 d_6 t^{12}}{1451520} + \frac{c_2 c_4 d_6 t^{12}}{806400} + \frac{c_1 c_5 d_6 t^{12}}{3628800} - \frac{c_6 d_6 t^{12}}{3628800}
\end{aligned}$$

In[70]:= SeriesCoefficient[%69, {t, 0, 6}]

Out[70]=

$$\begin{aligned}
& \frac{c_1^6}{5040} - \frac{c_1^4 c_2}{840} + \frac{11 c_1^2 c_2^2}{10080} + \frac{c_2^3}{1008} + \frac{c_1^3 c_3}{2016} + \frac{11 c_1 c_2 c_3}{10080} - \frac{c_3^2}{10080} - \frac{c_1^2 c_4}{2016} - \frac{c_2 c_4}{1120} - \\
& \frac{c_1 c_5}{5040} + \frac{c_6}{5040} - \frac{c_1^3 c_2 d_1}{1440} + \frac{1}{480} c_1 c_2^2 d_1 + \frac{c_1^2 c_3 d_1}{1440} - \frac{c_1 c_4 d_1}{1440} - \frac{c_1^4 d_1^2}{1440} + \\
& \frac{1}{360} c_1^2 c_2 d_1^2 + \frac{c_2^2 d_1^2}{480} + \frac{c_1 c_3 d_1^2}{1440} - \frac{c_4 d_1^2}{1440} + \frac{1}{144} c_1 c_2 d_1^3 + \frac{c_1^2 d_1^4}{288} + \frac{c_2 d_1^4}{288} + \\
& \frac{c_1 d_1^5}{240} + \frac{d_1^6}{720} + \frac{c_1^4 d_2}{720} - \frac{1}{180} c_1^2 c_2 d_2 - \frac{c_2^2 d_2}{240} - \frac{c_1 c_3 d_2}{720} + \frac{c_4 d_2}{720} - \frac{1}{48} c_1 c_2 d_1 d_2 - \\
& \frac{1}{72} c_1^2 d_1^2 d_2 - \frac{1}{72} c_2 d_1^2 d_2 - \frac{1}{48} c_1 d_1^3 d_2 - \frac{d_1^4 d_2}{120} + \frac{c_1^2 d_2^2}{144} + \frac{c_2 d_2^2}{144} + \frac{1}{48} c_1 d_1 d_2^2 + \\
& \frac{d_1^2 d_2^2}{80} - \frac{d_2^3}{360} + \frac{c_1 c_2 d_3}{48} + \frac{1}{72} c_1^2 d_1 d_3 + \frac{c_2 d_1 d_3}{72} + \frac{1}{48} c_1 d_1^2 d_3 + \frac{d_1^3 d_3}{120} - \frac{c_1 d_2 d_3}{48} - \\
& \frac{d_1 d_2 d_3}{60} + \frac{d_3^2}{240} - \frac{c_1^2 d_4}{72} - \frac{c_2 d_4}{72} - \frac{c_1 d_1 d_4}{48} - \frac{d_1^2 d_4}{120} + \frac{d_2 d_4}{120} + \frac{c_1 d_5}{48} + \frac{d_1 d_5}{120} - \frac{d_6}{120}
\end{aligned}$$

```
In[71]:= Expand[%70 /. {d1 -> e1 + 6 * t * H, d2 -> e2 + 5 * e1 * t * H + 15 * t^2 * H^2,
d3 -> e3 + 4 * e2 * t * H + 10 * e1 * t^2 * H^2 + 20 * t^3 * H^3,
d4 -> e4 + 3 * e3 * t * H + 6 * e2 * t^2 * H^2 + 10 * e1 * t^3 * H^3 + 15 * t^4 * H^4,
d5 -> e5 + 2 * e4 * t * H + 3 * e3 * t^2 * H^2 + 4 * e2 * t^3 * H^3 +
5 * e1 * t^4 * H^4 + 6 * t^5 * H^5, d6 -> e6 + e5 * t * H + e4 * t^2 * H^2 +
e3 * t^3 * H^3 + e2 * t^4 * H^4 + e1 * t^5 * H^5 + t^6 * H^6}]
```

Out[71]=

$$\begin{aligned}
& \frac{c1^6}{5040} - \frac{c1^4 c2}{840} + \frac{11 c1^2 c2^2}{10080} + \frac{c2^3}{1008} + \frac{c1^3 c3}{2016} + \frac{11 c1 c2 c3}{10080} - \frac{c3^2}{10080} - \frac{c1^2 c4}{2016} - \frac{c2 c4}{1120} - \\
& \frac{c1 c5}{5040} + \frac{c6}{5040} - \frac{c1^3 c2 e1}{1440} + \frac{1}{480} c1 c2^2 e1 + \frac{c1^2 c3 e1}{1440} - \frac{c1 c4 e1}{1440} - \frac{c1^4 e1^2}{1440} + \\
& \frac{1}{360} c1^2 c2 e1^2 + \frac{c2^2 e1^2}{480} + \frac{c1 c3 e1^2}{1440} - \frac{c4 e1^2}{1440} + \frac{1}{144} c1 c2 e1^3 + \frac{c1^2 e1^4}{288} + \frac{c2 e1^4}{288} + \\
& \frac{c1 e1^5}{240} + \frac{e1^6}{720} + \frac{c1^4 e2}{720} - \frac{1}{180} c1^2 c2 e2 - \frac{c2^2 e2}{240} - \frac{c1 c3 e2}{720} + \frac{c4 e2}{720} - \frac{1}{48} c1 c2 e1 e2 - \\
& \frac{1}{72} c1^2 e1^2 e2 - \frac{1}{72} c2 e1^2 e2 - \frac{1}{48} c1 e1^3 e2 - \frac{e1^4 e2}{120} + \frac{c1^2 e2^2}{144} + \frac{c2 e2^2}{144} + \frac{1}{48} c1 e1 e2^2 + \\
& \frac{e1^2 e2^2}{80} - \frac{e2^3}{360} + \frac{c1 c2 e3}{48} + \frac{1}{72} c1^2 e1 e3 + \frac{c2 e1 e3}{72} + \frac{1}{48} c1 e1^2 e3 + \frac{e1^3 e3}{120} - \frac{c1 e2 e3}{48} - \\
& \frac{e1 e2 e3}{60} + \frac{e3^2}{240} - \frac{c1^2 e4}{72} - \frac{c2 e4}{72} - \frac{c1 e1 e4}{48} - \frac{e1^2 e4}{120} + \frac{e2 e4}{120} + \frac{c1 e5}{48} + \frac{e1 e5}{120} - \frac{e6}{120} - \\
& \frac{1}{240} c1^3 c2 H t + \frac{1}{80} c1 c2^2 H t + \frac{1}{240} c1^2 c3 H t - \frac{1}{240} c1 c4 H t - \frac{1}{720} c1^4 e1 H t + \\
& \frac{1}{180} c1^2 c2 e1 H t + \frac{1}{240} c2^2 e1 H t + \frac{1}{720} c1 c3 e1 H t - \frac{1}{720} c4 e1 H t + \frac{1}{48} c1 c2 e1^2 H t + \\
& \frac{1}{72} c1^2 e1^3 H t + \frac{1}{72} c2 e1^3 H t + \frac{1}{48} c1 e1^4 H t + \frac{1}{120} e1^5 H t - \frac{1}{24} c1 c2 e2 H t - \\
& \frac{1}{24} c1^2 e1 e2 H t - \frac{1}{24} c2 e1 e2 H t - \frac{1}{12} c1 e1^2 e2 H t - \frac{1}{24} e1^3 e2 H t + \frac{1}{24} c1 e2^2 H t + \\
& \frac{1}{24} e1 e2^2 H t + \frac{1}{24} c1^2 e3 H t + \frac{1}{24} c2 e3 H t + \frac{1}{12} c1 e1 e3 H t + \frac{1}{24} e1^2 e3 H t - \\
& \frac{1}{24} e2 e3 H t - \frac{1}{12} c1 e4 H t - \frac{1}{24} e1 e4 H t + \frac{e5 H t}{24} - \frac{1}{240} c1^4 H^2 t^2 + \frac{1}{60} c1^2 c2 H^2 t^2 + \\
& \frac{1}{80} c2^2 H^2 t^2 + \frac{1}{240} c1 c3 H^2 t^2 - \frac{1}{240} c4 H^2 t^2 + \frac{1}{48} c1 c2 e1 H^2 t^2 + \frac{1}{48} c1^2 e1^2 H^2 t^2 + \\
& \frac{1}{48} c2 e1^2 H^2 t^2 + \frac{1}{24} c1 e1^3 H^2 t^2 + \frac{1}{48} e1^4 H^2 t^2 - \frac{1}{24} c1^2 e2 H^2 t^2 - \frac{1}{24} c2 e2 H^2 t^2 - \\
& \frac{1}{8} c1 e1 e2 H^2 t^2 - \frac{1}{12} e1^2 e2 H^2 t^2 + \frac{1}{24} e2^2 H^2 t^2 + \frac{1}{8} c1 e3 H^2 t^2 + \frac{1}{12} e1 e3 H^2 t^2 - \\
& \frac{1}{12} e4 H^2 t^2 + \frac{1}{24} c1 c2 H^3 t^3 + \frac{1}{72} c1^2 e1 H^3 t^3 + \frac{1}{72} c2 e1 H^3 t^3 + \frac{1}{24} c1 e1^2 H^3 t^3 + \\
& \frac{1}{36} e1^3 H^3 t^3 - \frac{1}{12} c1 e2 H^3 t^3 - \frac{1}{12} e1 e2 H^3 t^3 + \frac{1}{12} e3 H^3 t^3 + \frac{1}{48} c1^2 H^4 t^4 + \frac{1}{48} c2 H^4 t^4 + \\
& \frac{1}{48} c1 e1 H^4 t^4 + \frac{1}{48} e1^2 H^4 t^4 - \frac{1}{24} e2 H^4 t^4 + \frac{1}{40} c1 H^5 t^5 + \frac{1}{120} e1 H^5 t^5 + \frac{H^6 t^6}{120}
\end{aligned}$$



In[72]:= **Expand**[(1 + (a + b) \* t) \* (1 + (a + c) \* t) \*

(1 + (a + d) \* t) \* (1 + (b + c) \* t) \* (1 + (b + d) \* t) \* (1 + (c + d) \* t)]

Out[72]=

$$\begin{aligned}
 &1 + 3 a t + 3 b t + 3 c t + 3 d t + 3 a^2 t^2 + 8 a b t^2 + 3 b^2 t^2 + 8 a c t^2 + 8 b c t^2 + 3 c^2 t^2 + \\
 &8 a d t^2 + 8 b d t^2 + 8 c d t^2 + 3 d^2 t^2 + a^3 t^3 + 7 a^2 b t^3 + 7 a b^2 t^3 + b^3 t^3 + 7 a^2 c t^3 + \\
 &18 a b c t^3 + 7 b^2 c t^3 + 7 a c^2 t^3 + 7 b c^2 t^3 + c^3 t^3 + 7 a^2 d t^3 + 18 a b d t^3 + 7 b^2 d t^3 + \\
 &18 a c d t^3 + 18 b c d t^3 + 7 c^2 d t^3 + 7 a d^2 t^3 + 7 b d^2 t^3 + 7 c d^2 t^3 + d^3 t^3 + 2 a^3 b t^4 + \\
 &5 a^2 b^2 t^4 + 2 a b^3 t^4 + 2 a^3 c t^4 + 13 a^2 b c t^4 + 13 a b^2 c t^4 + 2 b^3 c t^4 + 5 a^2 c^2 t^4 + \\
 &13 a b c^2 t^4 + 5 b^2 c^2 t^4 + 2 a c^3 t^4 + 2 b c^3 t^4 + 2 a^3 d t^4 + 13 a^2 b d t^4 + 13 a b^2 d t^4 + \\
 &2 b^3 d t^4 + 13 a^2 c d t^4 + 30 a b c d t^4 + 13 b^2 c d t^4 + 13 a c^2 d t^4 + 13 b c^2 d t^4 + 2 c^3 d t^4 + \\
 &5 a^2 d^2 t^4 + 13 a b d^2 t^4 + 5 b^2 d^2 t^4 + 13 a c d^2 t^4 + 13 b c d^2 t^4 + 5 c^2 d^2 t^4 + 2 a d^3 t^4 + \\
 &2 b d^3 t^4 + 2 c d^3 t^4 + a^3 b^2 t^5 + a^2 b^3 t^5 + 3 a^3 b c t^5 + 7 a^2 b^2 c t^5 + 3 a b^3 c t^5 + a^3 c^2 t^5 + \\
 &7 a^2 b c^2 t^5 + 7 a b^2 c^2 t^5 + b^3 c^2 t^5 + a^2 c^3 t^5 + 3 a b c^3 t^5 + b^2 c^3 t^5 + 3 a^3 b d t^5 + \\
 &7 a^2 b^2 d t^5 + 3 a b^3 d t^5 + 3 a^3 c d t^5 + 15 a^2 b c d t^5 + 15 a b^2 c d t^5 + 3 b^3 c d t^5 + \\
 &7 a^2 c^2 d t^5 + 15 a b c^2 d t^5 + 7 b^2 c^2 d t^5 + 3 a c^3 d t^5 + 3 b c^3 d t^5 + a^3 d^2 t^5 + 7 a^2 b d^2 t^5 + \\
 &7 a b^2 d^2 t^5 + b^3 d^2 t^5 + 7 a^2 c d^2 t^5 + 15 a b c d^2 t^5 + 7 b^2 c d^2 t^5 + 7 a c^2 d^2 t^5 + \\
 &7 b c^2 d^2 t^5 + c^3 d^2 t^5 + a^2 d^3 t^5 + 3 a b d^3 t^5 + b^2 d^3 t^5 + 3 a c d^3 t^5 + 3 b c d^3 t^5 + \\
 &c^2 d^3 t^5 + a^3 b^2 c t^6 + a^2 b^3 c t^6 + a^3 b c^2 t^6 + 2 a^2 b^2 c^2 t^6 + a b^3 c^2 t^6 + a^2 b c^3 t^6 + \\
 &a b^2 c^3 t^6 + a^3 b^2 d t^6 + a^2 b^3 d t^6 + 2 a^3 b c d t^6 + 4 a^2 b^2 c d t^6 + 2 a b^3 c d t^6 + a^3 c^2 d t^6 + \\
 &4 a^2 b c^2 d t^6 + 4 a b^2 c^2 d t^6 + b^3 c^2 d t^6 + a^2 c^3 d t^6 + 2 a b c^3 d t^6 + b^2 c^3 d t^6 + \\
 &a^3 b d^2 t^6 + 2 a^2 b^2 d^2 t^6 + a b^3 d^2 t^6 + a^3 c d^2 t^6 + 4 a^2 b c d^2 t^6 + 4 a b^2 c d^2 t^6 + \\
 &b^3 c d^2 t^6 + 2 a^2 c^2 d^2 t^6 + 4 a b c^2 d^2 t^6 + 2 b^2 c^2 d^2 t^6 + a c^3 d^2 t^6 + b c^3 d^2 t^6 + \\
 &a^2 b d^3 t^6 + a b^2 d^3 t^6 + a^2 c d^3 t^6 + 2 a b c d^3 t^6 + b^2 c d^3 t^6 + a c^2 d^3 t^6 + b c^2 d^3 t^6
 \end{aligned}$$

In[73]:= **G6 = SeriesCoefficient**[%72, {t, 0, 6}]

Out[73]=

$$\begin{aligned}
 &a^3 b^2 c + a^2 b^3 c + a^3 b c^2 + 2 a^2 b^2 c^2 + a b^3 c^2 + a^2 b c^3 + a b^2 c^3 + a^3 b^2 d + \\
 &a^2 b^3 d + 2 a^3 b c d + 4 a^2 b^2 c d + 2 a b^3 c d + a^3 c^2 d + 4 a^2 b c^2 d + 4 a b^2 c^2 d + \\
 &b^3 c^2 d + a^2 c^3 d + 2 a b c^3 d + b^2 c^3 d + a^3 b d^2 + 2 a^2 b^2 d^2 + a b^3 d^2 + a^3 c d^2 + \\
 &4 a^2 b c d^2 + 4 a b^2 c d^2 + b^3 c d^2 + 2 a^2 c^2 d^2 + 4 a b c^2 d^2 + 2 b^2 c^2 d^2 + a c^3 d^2 + \\
 &b c^3 d^2 + a^2 b d^3 + a b^2 d^3 + a^2 c d^3 + 2 a b c d^3 + b^2 c d^3 + a c^2 d^3 + b c^2 d^3
 \end{aligned}$$

In[74]:= **G5 = SeriesCoefficient**[%72, {t, 0, 5}]

Out[74]=

$$\begin{aligned}
 &a^3 b^2 + a^2 b^3 + 3 a^3 b c + 7 a^2 b^2 c + 3 a b^3 c + a^3 c^2 + 7 a^2 b c^2 + 7 a b^2 c^2 + \\
 &b^3 c^2 + a^2 c^3 + 3 a b c^3 + b^2 c^3 + 3 a^3 b d + 7 a^2 b^2 d + 3 a b^3 d + 3 a^3 c d + \\
 &15 a^2 b c d + 15 a b^2 c d + 3 b^3 c d + 7 a^2 c^2 d + 15 a b c^2 d + 7 b^2 c^2 d + 3 a c^3 d + \\
 &3 b c^3 d + a^3 d^2 + 7 a^2 b d^2 + 7 a b^2 d^2 + b^3 d^2 + 7 a^2 c d^2 + 15 a b c d^2 + 7 b^2 c d^2 + \\
 &7 a c^2 d^2 + 7 b c^2 d^2 + c^3 d^2 + a^2 d^3 + 3 a b d^3 + b^2 d^3 + 3 a c d^3 + 3 b c d^3 + c^2 d^3
 \end{aligned}$$

In[75]:= **G4 = SeriesCoefficient**[%72, {t, 0, 4}]

Out[75]=

$$\begin{aligned}
 &2 a^3 b + 5 a^2 b^2 + 2 a b^3 + 2 a^3 c + 13 a^2 b c + 13 a b^2 c + 2 b^3 c + 5 a^2 c^2 + \\
 &13 a b c^2 + 5 b^2 c^2 + 2 a c^3 + 2 b c^3 + 2 a^3 d + 13 a^2 b d + 13 a b^2 d + 2 b^3 d + \\
 &13 a^2 c d + 30 a b c d + 13 b^2 c d + 13 a c^2 d + 13 b c^2 d + 2 c^3 d + 5 a^2 d^2 + \\
 &13 a b d^2 + 5 b^2 d^2 + 13 a c d^2 + 13 b c d^2 + 5 c^2 d^2 + 2 a d^3 + 2 b d^3 + 2 c d^3
 \end{aligned}$$

In[76]:= **G3 = SeriesCoefficient[%72, {t, 0, 3}]**

Out[76]=

$$a^3 + 7 a^2 b + 7 a b^2 + b^3 + 7 a^2 c + 18 a b c + 7 b^2 c + 7 a c^2 + 7 b c^2 + c^3 + 7 a^2 d + 18 a b d + 7 b^2 d + 18 a c d + 18 b c d + 7 c^2 d + 7 a d^2 + 7 b d^2 + 7 c d^2 + d^3$$

In[77]:= **G2 = SeriesCoefficient[%72, {t, 0, 2}]**

Out[77]=

$$3 a^2 + 8 a b + 3 b^2 + 8 a c + 8 b c + 3 c^2 + 8 a d + 8 b d + 8 c d + 3 d^2$$

In[78]:= **G1 = SeriesCoefficient[%72, {t, 0, 1}]**

Out[78]=

$$3 (a + b + c + d)$$

In[79]:= **SymmetricReduction[G6, {a, b, c, d}, {f1, f2, f3, f4}]**

Out[79]=

$$\{f_1 f_2 f_3 - f_3^2 - f_1^2 f_4, 0\}$$

In[80]:= **k6 = First[%79]**

Out[80]=

$$f_1 f_2 f_3 - f_3^2 - f_1^2 f_4$$

In[81]:= **SymmetricReduction[G5, {a, b, c, d}, {f1, f2, f3, f4}]**

Out[81]=

$$\{f_1 f_2^2 + f_1^2 f_3 - 4 f_1 f_4, 0\}$$

In[82]:= **k5 = First[%81]**

Out[82]=

$$f_1 f_2^2 + f_1^2 f_3 - 4 f_1 f_4$$

In[83]:= **SymmetricReduction[G4, {a, b, c, d}, {f1, f2, f3, f4}]**

Out[83]=

$$\{2 f_1^2 f_2 + f_2^2 + f_1 f_3 - 4 f_4, 0\}$$

In[84]:= **k4 = First[%83]**

Out[84]=

$$2 f_1^2 f_2 + f_2^2 + f_1 f_3 - 4 f_4$$

In[85]:= **SymmetricReduction[G3, {a, b, c, d}, {f1, f2, f3, f4}]**

Out[85]=

$$\{f_1^3 + 4 f_1 f_2, 0\}$$

In[86]:= **k3 = First[%85]**

Out[86]=

$$f_1^3 + 4 f_1 f_2$$

In[87]:= **SymmetricReduction[G2, {a, b, c, d}, {f1, f2, f3, f4}]**

Out[87]=

$$\{3 f_1^2 + 2 f_2, 0\}$$

In[88]:= **k2 = First[%87]**

Out[88]=

$$3 f_1^2 + 2 f_2$$

In[89]:= SymmetricReduction[G1, {a, b, c, d}, {f1, f2, f3, f4}]

Out[89]=  
{3 f1, 0}

In[90]:= k1 = First[%89]

Out[90]=  
3 f1

In[91]:= Expand[%71 /. {e1 → k1, e2 → k2, e3 → k3, e4 → k4, e5 → k5, e6 → k6}]

Out[91]=

$$\begin{aligned} & \frac{c1^6}{5040} - \frac{c1^4 c2}{840} + \frac{11 c1^2 c2^2}{10080} + \frac{c2^3}{1008} + \frac{c1^3 c3}{2016} + \frac{11 c1 c2 c3}{10080} - \frac{c3^2}{10080} - \frac{c1^2 c4}{2016} - \frac{c2 c4}{1120} - \\ & \frac{c1 c5}{5040} + \frac{c6}{5040} - \frac{1}{480} c1^3 c2 f1 + \frac{1}{160} c1 c2^2 f1 + \frac{1}{480} c1^2 c3 f1 - \frac{c1 c4 f1}{480} - \frac{c1^4 f1^2}{480} + \\ & \frac{1}{120} c1^2 c2 f1^2 + \frac{c2^2 f1^2}{160} + \frac{1}{480} c1 c3 f1^2 - \frac{c4 f1^2}{480} + \frac{1}{48} c1 c2 f1^3 + \frac{c1^2 f1^4}{96} + \frac{c2 f1^4}{96} + \\ & \frac{c1 f1^5}{80} + \frac{f1^6}{240} + \frac{c1^4 f2}{360} - \frac{1}{90} c1^2 c2 f2 - \frac{c2^2 f2}{120} - \frac{c1 c3 f2}{360} + \frac{c4 f2}{360} - \frac{1}{24} c1 c2 f1 f2 - \\ & \frac{1}{36} c1^2 f1^2 f2 - \frac{1}{36} c2 f1^2 f2 - \frac{1}{24} c1 f1^3 f2 - \frac{f1^4 f2}{60} + \frac{c1^2 f2^2}{72} + \frac{c2 f2^2}{72} + \\ & \frac{1}{24} c1 f1 f2^2 + \frac{f1^2 f2^2}{40} - \frac{f2^3}{180} - \frac{1}{72} c1^2 f1 f3 - \frac{c2 f1 f3}{72} - \frac{1}{24} c1 f1^2 f3 - \frac{f1^3 f3}{40} + \\ & \frac{f1 f2 f3}{120} + \frac{f3^2}{120} + \frac{c1^2 f4}{18} + \frac{c2 f4}{18} + \frac{c1 f1 f4}{6} + \frac{13 f1^2 f4}{120} - \frac{f2 f4}{15} - \frac{1}{240} c1^3 c2 H t + \\ & \frac{1}{80} c1 c2^2 H t + \frac{1}{240} c1^2 c3 H t - \frac{1}{240} c1 c4 H t - \frac{1}{240} c1^4 f1 H t + \frac{1}{60} c1^2 c2 f1 H t + \\ & \frac{1}{80} c2^2 f1 H t + \frac{1}{240} c1 c3 f1 H t - \frac{1}{240} c4 f1 H t + \frac{1}{16} c1 c2 f1^2 H t + \frac{1}{24} c1^2 f1^3 H t + \\ & \frac{1}{24} c2 f1^3 H t + \frac{1}{16} c1 f1^4 H t + \frac{1}{40} f1^5 H t - \frac{1}{12} c1 c2 f2 H t - \frac{1}{12} c1^2 f1 f2 H t - \\ & \frac{1}{12} c2 f1 f2 H t - \frac{1}{6} c1 f1^2 f2 H t - \frac{1}{12} f1^3 f2 H t + \frac{1}{12} c1 f2^2 H t + \frac{1}{12} f1 f2^2 H t - \\ & \frac{1}{12} c1 f1 f3 H t - \frac{1}{12} f1^2 f3 H t + \frac{1}{3} c1 f4 H t + \frac{1}{3} f1 f4 H t - \frac{1}{240} c1^4 H^2 t^2 + \\ & \frac{1}{60} c1^2 c2 H^2 t^2 + \frac{1}{80} c2^2 H^2 t^2 + \frac{1}{240} c1 c3 H^2 t^2 - \frac{1}{240} c4 H^2 t^2 + \frac{1}{16} c1 c2 f1 H^2 t^2 + \\ & \frac{1}{16} c1^2 f1^2 H^2 t^2 + \frac{1}{16} c2 f1^2 H^2 t^2 + \frac{1}{8} c1 f1^3 H^2 t^2 + \frac{1}{16} f1^4 H^2 t^2 - \frac{1}{12} c1^2 f2 H^2 t^2 - \\ & \frac{1}{12} c2 f2 H^2 t^2 - \frac{1}{4} c1 f1 f2 H^2 t^2 - \frac{1}{6} f1^2 f2 H^2 t^2 + \frac{1}{12} f2^2 H^2 t^2 - \frac{1}{12} f1 f3 H^2 t^2 + \\ & \frac{1}{3} f4 H^2 t^2 + \frac{1}{24} c1 c2 H^3 t^3 + \frac{1}{24} c1^2 f1 H^3 t^3 + \frac{1}{24} c2 f1 H^3 t^3 + \frac{1}{8} c1 f1^2 H^3 t^3 + \\ & \frac{1}{12} f1^3 H^3 t^3 - \frac{1}{6} c1 f2 H^3 t^3 - \frac{1}{6} f1 f2 H^3 t^3 + \frac{1}{48} c1^2 H^4 t^4 + \frac{1}{48} c2 H^4 t^4 + \\ & \frac{1}{16} c1 f1 H^4 t^4 + \frac{1}{16} f1^2 H^4 t^4 - \frac{1}{12} f2 H^4 t^4 + \frac{1}{40} c1 H^5 t^5 + \frac{1}{40} f1 H^5 t^5 + \frac{H^6 t^6}{120} \end{aligned}$$

In[92]:= **FunctionExpand**[1 - Binomial[6 - d, 6] + Binomial[5 - d, 6]]

Out[92]=

$$1 - \frac{1}{720} (-6 + d) (-5 + d) (-4 + d) (-3 + d) (-2 + d) (-1 + d) + \frac{1}{720} (-5 + d) (-4 + d) (-3 + d) (-2 + d) (-1 + d) d$$

In[93]:= **Expand**[(1/4) \* l1 \* l2 \* f1 +

(1/4) \* (l1^2 + l2) \* (f1^2 - 2 \* f2) + (1/2) \* l1 \* (f1^3 - 3 \* f1 \* f2 + 3 \* f3) + (1/4) \* (f1^4 - 4 \* f1^2 \* f2 + 4 \* f1 \* f3 + 2 \* f2^2) - 24 \* d + 24 \* %92]

Out[93]=

$$\frac{154 d}{5} - 45 d^2 + 17 d^3 - 3 d^4 + \frac{d^5}{5} + \frac{f1^4}{4} - f1^2 f2 + \frac{f2^2}{2} + f1 f3 + \frac{f1^3 l1}{2} - \frac{3 f1 f2 l1}{2} + \frac{3 f3 l1}{2} + \frac{f1^2 l1^2}{4} - \frac{f2 l1^2}{2} + \frac{f1^2 l2}{4} - \frac{f2 l2}{2} + \frac{f1 l1 l2}{4}$$

In[94]:= **Expand**[%93 /. {l1 -> (6 - d) \* H, l2 -> (d^2 - 6 \* d + 15) \* H^2, f1 -> 2 \* (d - 1) \* H, f2 -> (1/3) \* (d - 1) \* (5 \* d - 4) \* H^2, f3 -> (1/3) \* (d - 1)^2 \* (2 \* d - 1) \* H^3}]

Out[94]=

$$\frac{154 d}{5} - 45 d^2 + 17 d^3 - 3 d^4 + \frac{d^5}{5} - \frac{277 H^4}{9} + \frac{269 d H^4}{6} - \frac{299 d^2 H^4}{18} + \frac{8 d^3 H^4}{3} - \frac{d^4 H^4}{9}$$

In[95]:= **Expand**[%94 /. {H^4 -> d}]

Out[95]=

$$\frac{d}{45} - \frac{d^2}{6} + \frac{7 d^3}{18} - \frac{d^4}{3} + \frac{4 d^5}{45}$$

In[96]:= **Expand**[(1/d) \* %95]

Out[96]=

$$\frac{1}{45} - \frac{d}{6} + \frac{7 d^2}{18} - \frac{d^3}{3} + \frac{4 d^4}{45}$$

In[97]:= **Factor**[%96]

Out[97]=

$$\frac{1}{90} (-2 + d) (-1 + d) (-1 + 2 d) (-1 + 4 d)$$

In[98]:= **Expand[%91 / .**

{c1 → (8 - d) \* H, c2 → (d^2 - 8 \* d + 28) \* H^2, c3 → (56 - 28 \* d + 8 \* d^2 - d^3) \* H^3,  
 c4 → (d^4 - 8 \* d^3 + 28 \* d^2 - 56 \* d + 70) \* H^4,  
 c5 → (56 - 70 \* d + 56 \* d^2 - 28 \* d^3 + 8 \* d^4 - d^5) \* H^5,  
 c6 → (d^6 - 8 \* d^5 + 28 \* d^4 - 56 \* d^3 + 70 \* d^2 - 56 \* d + 28) \* H^6,  
 f1 → 2 \* (d - 1) \* H, f2 → (1 / 3) \* (d - 1) \* (5 \* d - 4) \* H^2,  
 f3 → (1 / 3) \* (d - 1)^2 \* (2 \* d - 1) \* H^3, f4 → (%97) \* H^4]}

Out[98]=

$$\begin{aligned} & \frac{518729H^6}{340200} + \frac{323dH^6}{120} + \frac{12323d^2H^6}{8100} + \frac{d^3H^6}{3} - \frac{709d^4H^6}{16200} - \frac{d^5H^6}{40} - \frac{403d^6H^6}{170100} + \frac{323H^6t}{60} + \\ & \frac{2339}{360}dH^6t + \frac{5}{2}d^2H^6t + \frac{13}{36}d^3H^6t - \frac{1}{30}d^4H^6t - \frac{1}{120}d^5H^6t + \frac{2339H^6t^2}{360} + \\ & \frac{11}{2}dH^6t^2 + \frac{17}{12}d^2H^6t^2 + \frac{1}{8}d^3H^6t^2 - \frac{1}{180}d^4H^6t^2 + \frac{11H^6t^3}{3} + \frac{19}{9}dH^6t^3 + \frac{1}{3}d^2H^6t^3 + \\ & \frac{1}{72}d^3H^6t^3 + \frac{19H^6t^4}{18} + \frac{3}{8}dH^6t^4 + \frac{1}{36}d^2H^6t^4 + \frac{3H^6t^5}{20} + \frac{1}{40}dH^6t^5 + \frac{H^6t^6}{120} \end{aligned}$$

In[99]:= **Expand[%98 / . {H^6 → d}]**

Out[99]=

$$\begin{aligned} & \frac{518729d}{340200} + \frac{323d^2}{120} + \frac{12323d^3}{8100} + \frac{d^4}{3} - \frac{709d^5}{16200} - \frac{d^6}{40} - \frac{403d^7}{170100} + \frac{323dt}{60} + \frac{2339d^2t}{360} + \\ & \frac{5d^3t}{2} + \frac{13d^4t}{36} - \frac{d^5t}{30} - \frac{d^6t}{120} + \frac{2339dt^2}{360} + \frac{11d^2t^2}{2} + \frac{17d^3t^2}{12} + \frac{d^4t^2}{8} - \frac{d^5t^2}{180} + \\ & \frac{11dt^3}{3} + \frac{19d^2t^3}{9} + \frac{d^3t^3}{3} + \frac{d^4t^3}{72} + \frac{19dt^4}{18} + \frac{3d^2t^4}{8} + \frac{d^3t^4}{36} + \frac{3dt^5}{20} + \frac{d^2t^5}{40} + \frac{dt^6}{120} \end{aligned}$$

In[100]:=

**Expand[%99 / . {t → 2 - 2 \* d + m}]**

Out[100]=

$$\begin{aligned} & \frac{30562169d}{340200} - \frac{1473d^2}{8} + \frac{611389d^3}{4050} - \frac{190d^4}{3} + \frac{229151d^5}{16200} - \frac{37d^6}{24} + \\ & \frac{2498d^7}{42525} + \frac{491dm}{4} - \frac{24419d^2m}{120} + \frac{2335d^3m}{18} - \frac{159d^4m}{4} + \frac{52d^5m}{9} - \frac{37d^6m}{120} + \\ & \frac{24419d^2m^2}{360} - \frac{175d^2m^2}{2} + \frac{163d^3m^2}{4} - \frac{65d^4m^2}{8} + \frac{26d^5m^2}{45} + \frac{175dm^3}{9} - \frac{55d^2m^3}{3} + \\ & \frac{50d^3m^3}{9} - \frac{13d^4m^3}{24} + \frac{55dm^4}{18} - \frac{15d^2m^4}{8} + \frac{5d^3m^4}{18} + \frac{dm^5}{4} - \frac{3d^2m^5}{40} + \frac{dm^6}{120} \end{aligned}$$

In[101]:=

**FunctionExpand[**  
**Binomial[7 + m, 7] - Binomial[m - d + 7, 7] - 3 \* Binomial[9 - 2 \* d + m, 7] +**  
**3 \* Binomial[9 - 3 \* d + m, 7] + 8 \* d \* Binomial[8 - 2 \* d + m, 6] - %100]**

Out[101]=

$$\begin{aligned}
 & -\frac{30\,562\,169\,d}{340\,200} + \frac{1473\,d^2}{8} - \frac{611\,389\,d^3}{4050} + \frac{190\,d^4}{3} - \frac{229\,151\,d^5}{16\,200} + \frac{37\,d^6}{24} - \frac{2498\,d^7}{42\,525} + \\
 & \frac{(-7+d-m)(-6+d-m)(-5+d-m)(-4+d-m)(-3+d-m)(-2+d-m)(-1+d-m)}{5040} + \\
 & \frac{1}{90} d(-8+2d-m)(-7+2d-m)(-6+2d-m)(-5+2d-m)(-4+2d-m)(-3+2d-m) + \\
 & \frac{1}{1680} (-9+2d-m)(-8+2d-m)(-7+2d-m)(-6+2d-m)(-5+2d-m) \\
 & (-4+2d-m)(-3+2d-m) - \frac{1}{1680} (-9+3d-m)(-8+3d-m)(-7+3d-m) \\
 & (-6+3d-m)(-5+3d-m)(-4+3d-m)(-3+3d-m) - \frac{491\,d\,m}{4} + \frac{24\,419\,d^2\,m}{120} - \\
 & \frac{2335\,d^3\,m}{18} + \frac{159\,d^4\,m}{4} - \frac{52\,d^5\,m}{9} + \frac{37\,d^6\,m}{120} - \frac{24\,419\,d\,m^2}{360} + \frac{175\,d^2\,m^2}{2} - \frac{163\,d^3\,m^2}{4} + \\
 & \frac{65\,d^4\,m^2}{8} - \frac{26\,d^5\,m^2}{45} - \frac{175\,d\,m^3}{9} + \frac{55\,d^2\,m^3}{3} - \frac{50\,d^3\,m^3}{9} + \frac{13\,d^4\,m^3}{24} - \frac{55\,d\,m^4}{18} + \frac{15\,d^2\,m^4}{8} - \\
 & \frac{5\,d^3\,m^4}{18} - \frac{d\,m^5}{4} + \frac{3\,d^2\,m^5}{40} - \frac{d\,m^6}{120} + \frac{(1+m)(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}{5040}
 \end{aligned}$$

In[102]:=

**p0 = Expand[%101 /. {m -> 0}]**

Out[102]=

$$-\frac{2\,303\,699\,d}{340\,200} + \frac{3109\,d^2}{90} - \frac{133\,637\,d^3}{2025} + \frac{2179\,d^4}{36} - \frac{456\,221\,d^5}{16\,200} + \frac{1157\,d^6}{180} - \frac{24\,368\,d^7}{42\,525}$$

In[103]:=

**Factor[p0]**

Out[103]=

$$\frac{(-1+d)d(-1+2d)(2\,303\,699 - 4\,840\,923d + 3\,320\,849d^2 - 947\,157d^3 + 97\,472d^4)}{340\,200}$$

In[104]:=

**p1 = Expand[%101 /. {m -> 1}]**

Out[104]=

$$-\frac{4\,034\,939\,d}{340\,200} + \frac{2617\,d^2}{45} - \frac{848\,873\,d^3}{8100} + \frac{3185\,d^4}{36} - \frac{595\,631\,d^5}{16\,200} + \frac{1327\,d^6}{180} - \frac{24\,368\,d^7}{42\,525}$$

In[105]:=

**Factor [p1]**

Out[105]=

$$\frac{(-1+d) d (-1+2 d) (4034939 - 7679703 d + 4543679 d^2 - 1107807 d^3 + 97472 d^4)}{340200}$$

In[106]:=

**p2 = Expand [%101 /. {m → 2}]**

Out[106]=

$$-\frac{6454139 d}{340200} + \frac{2713 d^2}{30} - \frac{315887 d^3}{2025} + \frac{493 d^4}{4} - \frac{752681 d^5}{16200} + \frac{499 d^6}{60} - \frac{24368 d^7}{42525}$$

In[107]:=

**Factor [p2]**

Out[107]=

$$\frac{(-1+d) d (-1+2 d) (6454139 - 11403003 d + 5951729 d^2 - 1268457 d^3 + 97472 d^4)}{340200}$$

In[108]:=

**DZ = (d / 3) \* (d - 1) ^ 2 \* (2 \* d - 1)**

Out[108]=

$$\frac{1}{3} (-1+d)^2 d (-1+2 d)$$

In[109]:=

**KH2 = Expand [4 \* p1 - 2 \* p2 - 2 \* p0 + 2 \* DZ]**

Out[109]=

$$\frac{152 d}{45} - \frac{44 d^2}{3} + \frac{193 d^3}{9} - \frac{37 d^4}{3} + \frac{98 d^5}{45}$$

In[110]:=

**Factor [%109]**

Out[110]=

$$\frac{1}{45} (-1+d) d (-1+2 d) (152 - 204 d + 49 d^2)$$

In[111]:=

**K2HHc2 = Expand [12 \* p1 - 12 \* p0 - 2 \* DZ + 3 \* KH2]**

Out[111]=

$$-\frac{754 d}{15} + \frac{710 d^2}{3} - 398 d^3 + 297 d^4 - \frac{1451 d^5}{15} + \frac{34 d^6}{3}$$

In[112]:=

**Factor[%111]**

Out[112]=

$$\frac{1}{15} (-1 + d) d (-1 + 2 d) (-754 + 1288 d - 598 d^2 + 85 d^3)$$

In[113]:=

**Hc2 = -(2/3) \* (2\*d - 5) \* (5\*d - 19) \* DZ + (4\*d - 11) \* KH2**

Out[113]=

$$-\frac{2}{9} (-1 + d)^2 d (-5 + 2 d) (-1 + 2 d) (-19 + 5 d) +$$

$$(-11 + 4 d) \left( \frac{152 d}{45} - \frac{44 d^2}{3} + \frac{193 d^3}{9} - \frac{37 d^4}{3} + \frac{98 d^5}{45} \right)$$

In[114]:=

**Factor[%113]**

Out[114]=

$$\frac{1}{45} (-1 + d) d (-1 + 2 d) (-722 + 1272 d - 625 d^2 + 96 d^3)$$

In[115]:=

**K2H = K2HHc2 - Hc2**

Out[115]=

$$-\frac{754 d}{15} + \frac{710 d^2}{3} - 398 d^3 + 297 d^4 - \frac{1451 d^5}{15} +$$

$$\frac{34 d^6}{3} + \frac{2}{9} (-1 + d)^2 d (-5 + 2 d) (-1 + 2 d) (-19 + 5 d) -$$

$$(-11 + 4 d) \left( \frac{152 d}{45} - \frac{44 d^2}{3} + \frac{193 d^3}{9} - \frac{37 d^4}{3} + \frac{98 d^5}{45} \right)$$

In[116]:=

**Factor[%115]**

Out[116]=

$$\frac{1}{45} (-1 + d) d (-1 + 2 d) (-10 + 3 d) (154 - 213 d + 53 d^2)$$



In[117]:=

$$\text{Kc2} = -(2/3) * (2 * d - 5) * (5 * d - 19) * \text{KH2} + (4 * d - 11) * \text{K2H}$$

Out[117]=

$$-\frac{2}{3} (-5 + 2d) (-19 + 5d) \left( \frac{152d}{45} - \frac{44d^2}{3} + \frac{193d^3}{9} - \frac{37d^4}{3} + \frac{98d^5}{45} \right) +$$

$$(-11 + 4d) \left( -\frac{754d}{15} + \frac{710d^2}{3} - 398d^3 + 297d^4 - \frac{1451d^5}{15} + \frac{34d^6}{3} + \frac{2}{9} (-1 + d)^2 d (-5 + 2d) \right)$$

$$(-1 + 2d) (-19 + 5d) - (-11 + 4d) \left( \frac{152d}{45} - \frac{44d^2}{3} + \frac{193d^3}{9} - \frac{37d^4}{3} + \frac{98d^5}{45} \right)$$

In[118]:=

**Factor [%117]**

Out[118]=

$$\frac{1}{135} (-1 + d) d (-1 + 2d) (21940 - 46104d + 31627d^2 - 9021d^3 + 928d^4)$$

In[119]:=

**Expand [24 \* p0 + Kc2]**

Out[119]=

$$\frac{d}{14175} - \frac{d^3}{675} + \frac{4d^5}{675} - \frac{64d^7}{14175}$$

In[120]:=

**Factor [%119]**

Out[120]=

$$\frac{(-1 + d) d (1 + d) (-1 + 2d) (1 + 2d) (-1 + 4d) (1 + 4d)}{14175}$$